1st MATHEMATICAL CONTEST of Friendship in Honor and Memory OF GRAND DUCHY OF LITHUANIA

1. The natural number N is a multiple of 2009 and the sum of its (decimal) digits equals 2009.

   (A) Find one such number.

   (B) Find the smallest such number.

2. Let

$$f(x) = ax^3 + bx^2 + cx + d$$

be a polynomial with real coefficients. Given that $f(x)$ has three real positive roots and that $f(0) < 0$, prove that

$$2b^3 + 9a^2 d - 7abc \leq 0.$$

3. Solve the equation

$$x^2 + 2 = 4\sqrt{x^3 + 1}.$$

4. A triangle ABC has an obtuse angle at B. The perpendicular at B to AB meets AC at D, and $CD = AB$. Prove that

$$AD^2 = AB \cdot BC$$

if and only if

$$\angle \mathrm{CBD} = 30^\circ.$$

5. Consider a table whose entries are integers. Adding a same integer to all entries on a same row, or on a same column, is called an *operation*. It is given that, for infinitely many positive integers n, one can obtain, through a finite number of operations, a table having all entries divisible by n. Prove that, through a finite number of operations, one can obtain the table whose all entries are zeroes.