The Instationary Navier-Stokes System on Exterior Domains in Weighted Function Spaces

Reinhard Farwig

Technische Universität Darmstadt, Darmstadt, Germany

We consider an approach to the time-periodic (Navier-)Stokes system and the stability problem for stationary solutions on an exterior domain $\Omega \subset \mathbb{R}^n$, $n \geq 3$, with boundary condition $u = 0$ on $\partial\Omega$ in weighted function spaces $L^q_s(\Omega)$. Here the space $L^q_s(\Omega)$ is equipped with the norm $u \mapsto \|u(1+|x|^2)^{s/2}\|_{L^p}$ where $-\frac{n}{p} < s < \frac{n}{p'}$ so that the analysis is achieved within the theory of Muckenhoupt weights.

The main part concerns global-in-time estimates of the classical Stokes operator $A = -P\Delta$ and a perturbed Stokes operator defined by $A_w u = Au + u \cdot \nabla w + w \cdot \nabla u$ based on a stationary solution w of type $|w(x)| \leq c|x|^{-(n-2)}$. These results are used to construct time-periodic solutions in critical, weighted function spaces when $n \geq 4$ and to solve the stability problem when $n \geq 3$.

The talk is based on joint papers [1, 2] with K. Tsuda (Kyushu Sangyo University, Fukuoka).

References

[1] R. Farwig and K. Tsuda: The time periodic problem for the Navier-Stokes equations in exterior domains in weighted spaces. arXiv:2409.17590

[2] R. Farwig and K. Tsuda: The critial decay rate of stability for stationary solutions to the Navier-Stokes equations in exterior domains. Manuscript (2025)