APPROVED
By the committee of doctoral studies in mathematics
at 2014-05-02,
protocol No. 2(22)

RULES FOR THE ADMISSION TO THE DOCTORAL STUDIES IN MATHEMATICS

The exam is taken in the form of an interview.
The applicant presents his master thesis or other scientific result (by using slides or writing on the blackboard). Duration of presentation is up to 10 minutes. After presentation, the examiners can ask some questions related to the program given below and discuss with the applicant the subject of his/her prospective PhD thesis. Parts I, II and III are compulsory for all participants, while each applicant must choose between IVA and IVB.

PROGRAM

I. Mathematical Analysis

1. Limits of sequences and functions. Theorems about limits of monotonic sequences and functions. Examples.
2. Continuous functions. Weierstrass and Bolzano-Cauchy's theorems. Examples.
3. Uniformly continuous functions. Examples. Cantor's theorem.
4. Taylor's formula with various remainder terms. Examples.
5. Extrema for functions of one and several variables. Examples.
6. Definition and properties of the integral of a function of one and several variables. Continuity of the function defined by a definite integral and its derivative. Newton's formula.
7. Convergence tests. Examples. Dirichlet's and Riemann's theorems on rearrangements of number series.
8. Uniform convergence of the sequences of functions and series. Examples. Theorems on differentiation and integration of the function series term-by-term.
9. Indefinite integrals. Their comparison. Examples.
10. Theorem about the reduction of a double integral into iterated integral (Fubini's theorem). Examples.
11. Theorem about the change of variable in multiple integrals. Examples.

II. Geometry

1. Properties and geometric interpretations of the scalar multiplication of vectors, vector multiplication and the scalar triple product.
2. Equations of lines and planes.
3. Definitions and canonical equations of ellipses, hyperbolas and parabolas. Second-order curves and surfaces.

III. Algebra

1. Matrices, matrix operations.
2. Inverse of a matrix, its calculation.
3. Systems of linear equations, Cramer's rule.
4. Kronecker-Capelli theorem.
5. Homogeneous system of linear equations, properties of its solutions.
6. Definition of a linear space, examples. Linear independence of vectors.
7. Group, subgroup, cyclic groups, examples.

IVA. Probability Theory and Mathematical Statistics

1. Probability axioms.
2. Distribution function of a random variable and its properties. The mean and the variance of a random variable, their properties and examples.
3. Binomial, Poisson and normal distributions, their means and variances. Approximation theorems.
4. The weak law of large numbers, Chebyshev's theorem and its corollaries.
5. Central limit theorem, Lindeberg's condition.
6. Estimation methods: method of moments estimator (examples), maximum likelihood estimator (examples).
7. Confidence intervals. Confidence intervals for a sample mean and variance.
8. Statistical hypothesis testing, type I and type II errors. Normality tests.

IVB. Differential Equations

1. Normal system of differential equations, its solution, Cauchy's problem. Existence and uniqueness theorem for the normal system of differential equations.
2. Linear homogeneous system of differential equations. Cauchy's problem. Properties of the system (formulation). Linear dependence of solutions.
3. The Wronskian of linear homogeneous system of differential equations and its properties (formulation). Fundamental system of solutions and general solution (definition).
4. Linear inhomogeneous system of differential equations and its general solution structure. The method of variation of parameters.
5. First integral of normal system of differential equations (definition). Analytical property of the first integral.
6. Classification of partial differential equations of the second order.