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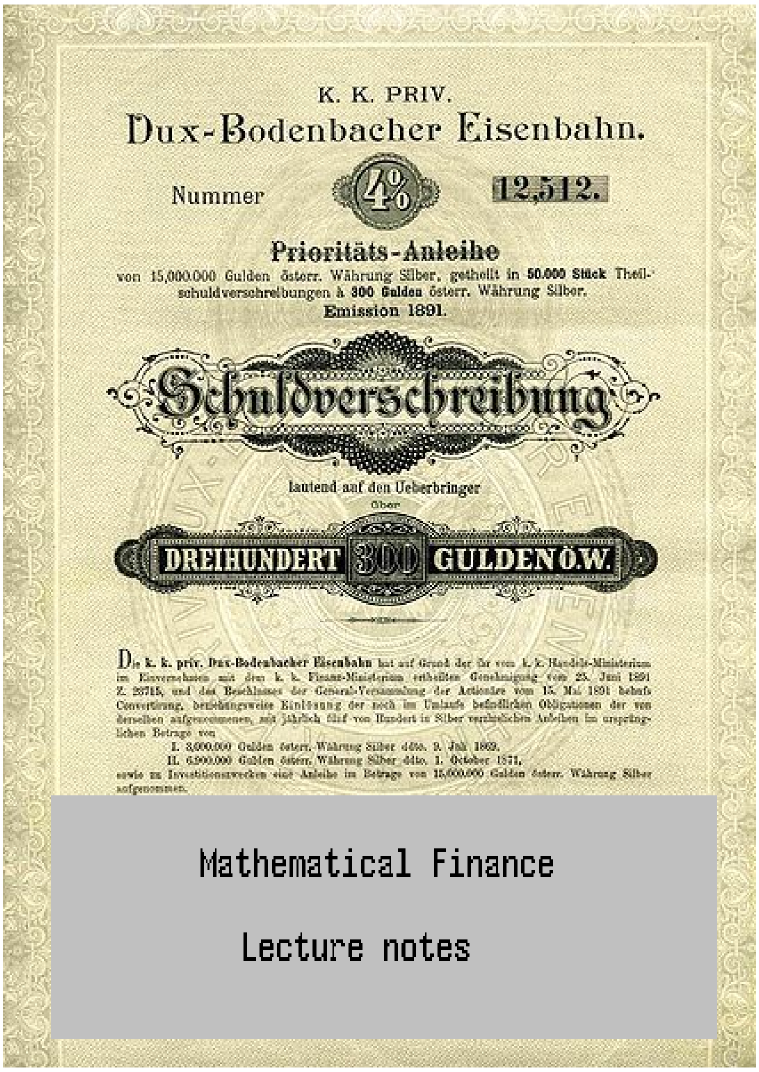

Mathematical Finance

Lecture notes

CONTENT

Chapter 1. SIMPLE AND COMPOUND INTEREST

Chapter 2. CASH FLOWS

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Chapter 1. SIMPLE AND COMPOUND INTEREST

Objectives:

• To repeat financial concepts which are to be applied within the main chapters.

• To simulate the mathematical and real content situations, while assessing these situations based on the numerical arguments.

An introductory chapter will briefly review the basic concepts that are to be operated within this entire publication. At the end of the chapter a set of tasks with answers will be provided in order a reader would be able to test the acquired knowledge.

Note While referring to the financial resources (money), we will indicate only a face value of money, and not a specific currency. For example, we will write the sum of LTL 220 briefly as 220.

Note The answers to the tasks and examples will often be inaccurate numbers; however, when typing answers we will usually use the equality sign at the accuracy of an error. A reader is warned that very often answers are rounded to the unit.

1.1 Simple interest

In the economy, the concept of capital is not uniquely defined, although it is generally agreed that these are the means of creating the added value. Thus, using the concept of capital in any context, assumptions need to be made. In this paper, we will identify the concept of capital as financial assets or, in other words, as financial resources.

The moment in time, which is considered the starting point for the performed calculations, will be called a focal date, or the focal date point. A capital will be called the present value at the focal date point. The present value will be marked by the letter P. In other words, the present value is the financial resources at the focal date point.

A value P in the future will be called the *present* or *principal* and will be marked by the letter S sometimes this letter will be typed with an index.

Note Sometimes the present value is called *maturity*.

Note During the examination of the issues of asset values at different points in time, very important terms of an actual value and a nominal value are faced.

A nominal asset (capital) value will be identified to a cash value of this asset. Meanwhile, an actual value is a relative ratio, in other words, it is the value at the analyzed point of time which is compared with the value at the other point of time.

Interest will be called the amount of money (at its nominal value), by which the present value differs from the maturity value after a certain interval of time. An interest is marked by the letter I. Based on this definition, we obtain that $I = S - P$. Thus, an interest characterizes capital gains at the fixed interval of time. Depending on the time interval an interest can be called a monthly interest, a semi-annual interest, annual interest, etc. In other words, an interest is the fee for the use of money.

Definition The process during which the accrued interest is calculated will be called the conversion. The time interval after which the conversion is performed is called the interest period or very often the interest conversion period as well.

Note We want to note that very often interest means the absolute capital gains, while comparing the future and fixed capital in terms of the actual (real) value, these two capital values may be identical or the future value may be lower than the fixed capital.

Definition Suppose that at the end of an interest period, the interest is capitalized if this interest is added to the fixed capital.

Note During the conversion an interest may be added to the capital, but it may also be left uncapitalised, i.e. it may be excluded from the subsequent process of accumulation.

Definition Interest rate (marked as r) is called the interest-initial capital ratio within a set period of time, which is called an interest period (interval). We have that

$$r = \frac{I}{P}.$$

An interest period is called the interval of time, for which the official (unambiguously understandable) interest rate is indicated. We will assume below that the said interest period is one year, as result, no other cases are to be analyzed. While indicated an interest rate in this publication we will use the general consensus that this period is one year. Thus, in the case it is said that the bank interest rate is 8%, it is understood that it is the annual interest rate. The use of this interest rate in the actual calculations will depend on the rate calculation methodology. It is easy to understand that an interest defines the absolute capital gains, while an interest rate means a relative (and interest bearing) capital increase after the conversion period.

We will note that a conversion period and an interest period do not necessarily coincide. An interest rate per conversion period is called the *actual interest rate.*

Definition Discounting is called the process, during which the present capital value is determined in respect of the future value.

In other words, during discounting we set the values of $S(t)$ at the period of time t, the actual value at the period of time $t_0; t_0 < t$. This discounted value we denote by $P(t_0) =: P$ or D.

Thus, the conversion is the method for the determination of the maturity value of an initial capital (or the comparison of the capital value with its maturity value), while discounting is the comparison of the maturity value with the value at the previous period of the time.

A discount rate (marked by d) is called the interest-future capital ratio

$$d = I/S.$$

This discount rate is sometimes known as the actual discount rate of the interest period. If the interest rates are established during the period of one year, this rate is called simply a discount rate.

Note Ordinary, when is not said before, talking about interest rate or discount rate we have in mind *yearly interest rate* and *yearly discount rate.* If we use notation *actual interest or actual discount rates* we mean that these rates are for some interest period which ordinary smaller then year.

During the conversion the accrued interest is determined, which can be calculated in the following way: $I = rP$, and after discounting, the future interest rates are calculated using the formula $I = dS$.

Using the defined concepts we can establish a link between an actual interest rate and a discount rate. In other words, we will specify the link, by which a discount and an interest rates at the same period are associated. We suppose that the actual (period) interest rate is i. Thus, at the end of the interest conversion period we receive the future value of

$$S = (1 + i)P.$$

On the other hand, if the actual discount rate (of the same long-term discounting period) is d, based on the discount definition we receive that

$$P = (1 - d)S.$$

Of the latter two equalities, which are compared in respect of S, we obtain that $1 + i = \frac{1}{1-d}$. Having solved the value of d in relation to i, or vice versa, we obtain the following functional relationship between a discount and an interest rate during the same time interval:

$$i = \frac{d}{1-d}, \quad d = \frac{i}{1+i}$$

We see that an interest rate is the discount function $i = f(d)$ and vice versa, $d = g(i)$.

Note The interest rate and discount rate indicated in the formulas above both are rates of the same converse period.

1.2 The capital changes in the case of a simple interest

We will assess the formulas for the calculation of the present value and the maturity value of a capital.

Let us assume that the initial capital is P and the actual interest rate (of a certain time interval) is i. Then the capital of the end of the period is equal to $S = P + iP = P(1 + i)$.

Note Assume that the annual interest rate is r. Let $t \in [0, 1]$. Then, any actual interest rate can be expressed in the following way: $i = tr$, where t indicates the interest-bearing part of a year for the interest period. It is easy to understand that if the interest is converted m times a year, when $i = \frac{r}{m}$.

Definition An interest will be called a simple interest, if at the end of each conversion period this interest is fixed and calculated from an initial capital, i.e. an interest is not capitalized.

Let us mark the capital by P_k at the $k-$ th period of time (after the k conversion periods), assuming that $k \in N$. The principal as usual we denote by P, and the actual interest rate (an interest rate per conversion period) is i. After the first conversion period, the maturity value of the principal is $P_1 = P + iP = P(1 + i)$, after the second conversion period it is $P_2 = P_1 + iP = P(1 + 2i)$, etc., after the k-th conversion period,

$$P_k = P_{k-1} + iP = P(1 + ki).$$

The formula

$$S = P_k = P(1 + ki)$$

is called a *compound formula* in case simple interest, or alternatively, a formula for the future value calculation, in the case of a simple interest, where time is discrete. The method obtaining future value having principal is called *compounding*.

Let us note that the sequence $\{P_k\}$ is an arithmetic progression. The denominator of the progression is equal to iP. While solving the discounting task in the case of a simple interest we will obtain that

$$P = \frac{S}{1 + ik}, \tag{1}$$

here k is the number of discount periods. This method ordinary is called *discounting*.

The ratio
$$\nu := \frac{1}{(1+ik)}$$
is called a discount factor in the case of a simple interests. We will draw the attention of a reader to the fact that in this case discounting is performed in respect of the entire accumulated value of an initial capital, and is applied to all the periods of k conversion.

Let us analyse the discounting task in a slightly different manner.

A simple discount, applicable to the intervals of k conversion, will be called the *discounting method*, when the discounted value during each of the conversion periods is calculated of the same maturity value of a capital, while discounting for the number of periods k, which is the number of conversion periods between the moments of discounting and the final values. Suppose that $P_k = S$ and actual discount rate (discount rate for the conversion period) is d. Then $P_{k-1} = S - S \cdot d = S(1-d)$. Further, $P_{k-2} = S(1 - 2 \cdot d)$, *etc.* $P = S(1 - k \cdot d)$, if k could be noticed during the discounting period. Thus, the major formula of a simple discount can be written as follows:
$$P = S(1 - kd). \tag{2}$$
We have found a correlation between an interest rate and a discount rate in one conversion period.

We find a correlation between the interest rate of the period and the discount rate of the period, when the number of periods remaining to the end of accumulation (discounting) is k. Let us align the relations (1) and (2) in respect of the fixed capital. We obtain that
$$1 - kd = \frac{1}{1+ik}.$$
Two correlations are obtained from the last equality in the case of a simple interest:
$$d = \frac{i}{1+ik}, \quad i = \frac{d}{1-kd}.$$
We will note that both i and d is an actual (period) rates rather than the rates of the entire period. We see that if the period interest rate is i, in order to determine the discount rate during the conversion periods k we have to divide this period rate by the value larger than 1. Thus, the growth of the number of conversion periods results in the decrease in the discount rate for the period (at a constant interest rate of the period). The interest rate satisfies the reverse relationship. I.e. if we know the discount rate of the period, which is constant, and if we know the number of periods during which the future value should be accumulated, the rate of the period increases with the increasing number of periods. We are going to consider the case of s simple interest, where time is a continuous variable denoting the number of years (not necessarily a whole number).

Let us summarize the above analysed formulas for any period of time, i.e. when the conversion is performed at any period of time. At the continuous time during the calculation of a future value (during discounting), we assume the actual interest rate as an annual interest rate. An annual interest rate will be marked by r.

We assume that the interest is converted for m times during a year, while the interest rate of a period is i. If the annual interest rate is r, the actual rate is $i = r/m$. In addition, if the time interval during which a capital is invested is t years and it is a multiple of m (in this case not necessarily a whole number), then $k = mt$. Then taking the relations

$$S = (1 + ik)P = (1 + \frac{r}{m} \cdot mt)P$$

into account, it follows that we can calculate the future value in the following way:

$$S = (1 + rt)P. \text{ moreover } \quad r = \frac{S - P}{P}.$$

It is known that $I = rtP$. The discount formula in this case is as follows:

$$P = \frac{S}{1 + rt}. \tag{3}$$

Furthermore

$$d = \frac{r}{1 + rt}, \quad r = \frac{d}{1 - dt}.$$

Note We would like to note that in this case time t is usually expressed by the parts of a year, while the given interest rate is an annual rate. Thus, in this case, time (days, months, etc.) has to be expressed by the parts of a year. Usually, while performing financial operations the number of days in a year is agreed.

Note If it is not separately mentioned otherwise, we assume that the number of days in a year is 365. During the calculation of interest between any two fixed dates, we usually assume that the first day is not included in the rate range, while the last is included, but it may be vice versa, as well. We would like to note that from the results of a calculation do not depend on this.

Note We will assume that the number of days of February is equal to 28 and that of a year is 365. Then, the time interval between November 12 and May 2 will include 174 days, while t = time is expressed in years.

If the values of P and r is known, the simple interest formula indicates a linear link between S and time t. That is to say

$$S = kt + b,$$

a rate of the linear function direction $k = rP$, and the free member $b = P$. On the other hand, the same task can be solved by using a discount rate. From the last equality we obtain that

$$P = \frac{S}{1 + rt} = \Big(1 - (\frac{r}{1 + rt}) \cdot t\Big)S = (1 - dt)S.$$

As we can see, if the future value is known, we find the present value when a period and an interest rate are known. In this case r and d is yearly interest and discount rates.

1.3 Equivalent values in case simple interest

We discuss about the situation when value of the principle P changes in time to the future value, which depend from a duration in time and the interest rate r. Thus the value of principle at any particular point is a *dated value* and the various dated values at different points in time linear depend to the principal value.

It is clear that due to the time value of money, sums of money located at different point in time are not directly comparable. Given a choice between 1000 today and 1500 after ten years. It is does not follow, that a large sum is preferable.

To compare two different sums we must select some time point, which allows us to compare different amounts in different time intervals. This comparison date is called *focal date*. It is used to obtain the dated value of the sums of money at a specific point in time.

Date value at the same point in time is directly comparable using formulas: amount formula

$$S = (1 + rt)P$$

or present value formula

$$P = \frac{S}{1 + rt}.$$

The choice of formula for finding focal date depends on the due date, before or after focal date is due date. In fig. 1 graphically are both positions shown of the due date.

1) If the due date falls before the focal date, use the amount formula (fig. 1. above);

2) if the due date falls after the focal date, use the present value formula.

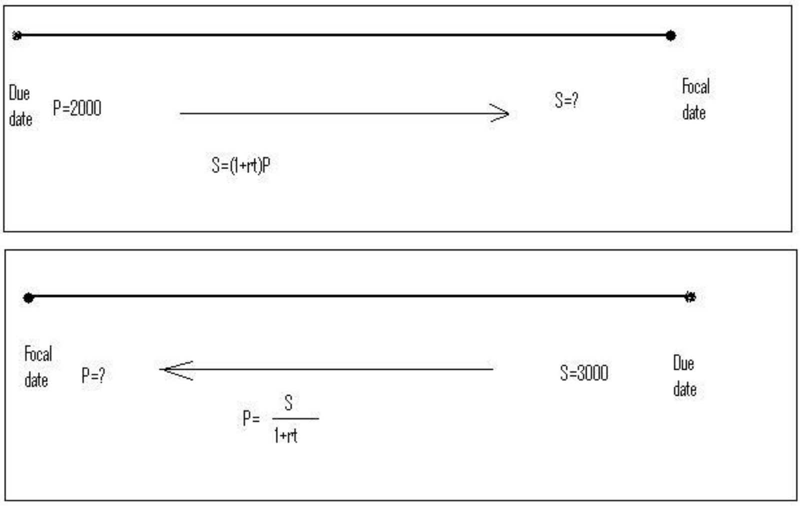

fig. 1.

Example A debt can be paid off by payments of 92000 one year from now and 130000 two years from now. Determine the single payment now which would settle the debt allowing for simple interest at 15%.

In this example the focal date (now) is earlier relative to the both given dates, in both cases we apply present value formula to obtain present values P_1 and P_2. Thus we have:

$$P_1 = \frac{92000}{(1 + 0.15)} = 80000, \quad P_2 = \frac{130000}{1 + 0.15 \cdot 2} = 100000.$$

Single payment now equals to 180000.

Consider example when focal date separates different due dates.

Example Debt payment of 400 due today, 500 due in 5 months and 618 due in one year, are to be combined into a single payment to be made 9 months from now with interest allowed at 12%.

Considering this situation we see, that two due dates are before focal date and one due date -after. Therefore, the focal date is in future time relative to the 400 and 500 amounts. Thus for both amounts we apply formula $S = (1+rt)P$. The amount 400 at the focal date equals to

$$S_1 = (1 + rt)P = (1 + 0.12 \cdot \frac{9}{12}) = 436;$$

and amount 500 at the focal date equals to

$$S_2 = (1 + 0.12 \cdot \frac{4}{12}) = 520.$$

The focal date is earlier relative to the 618 thus we apply present value formula

$$P = \frac{S}{1+rt} = \frac{618}{1 + 0.12 \cdot \frac{3}{12}} = 600.$$

Single payment required in 9 months from now equals $L = 436 + 520 + 600 = 1556$.

Consider example which is related so-called *equation of values* at the focal date.

Example Debts of 40000 due now and 70000 due in 5 month are to be settled by a payment of 50000 in 3 months and a final payment in 8 months. Determine the value of the final payment at 15%, with focal date 8 month from now.

The final value is unknown set it by x.

Graphical representation of the problem is given in fig 2..

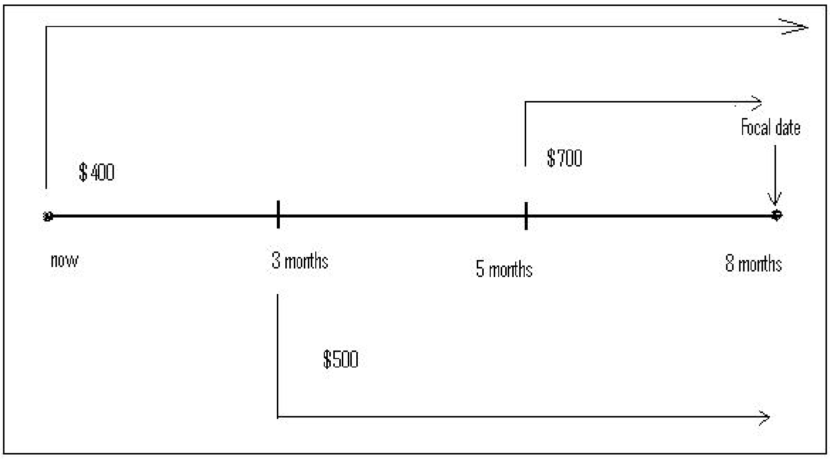

fig 2.

Thus we have the following equation of values:

$$(1 + 0.15 \cdot \frac{5}{12})500 + x = (1 + 0.15 \cdot \frac{8}{12})400 + (1 + 0.15 \cdot \frac{3}{12})700.$$

Solving this equation we obtain that the final payment to be made in 8 months is 635.

Sometimes, it is necessary to calculate the present value P, given the compound amount. A formula for P may be derive by equality (2):

$$P = (1+i)^{-n}S = \frac{S}{(1+i)^n}.$$

The last formula gives the principal P which must be invested at the rate of i per version period for n conversion periods so that the compound amount is S. As above we call P the present value of S.

Example What sum of money should be invested for 4 years at 9 percent compounded monthly in order to provide a compound of 8000?

$i = \frac{0.09}{12} = 0.0075,\ n = mt = 12 \cdot 4 = 48$. Then

$$P = \frac{S}{(1+i)^n} = \frac{8000}{(1+0.0075)^4 8000} = 5588.91.$$

Example A trust fund for child education is being set up by a single payment so that at the end of 15 years there will be 24000. If the fund earns interest at the rate of 7 persent compounded semiannually, how much money should be paid into the fund initially?

We want the present value of 24000 due in 15 years. Thus $S = 24000,\ i = \frac{0.07}{2} = 0.035$, and $n = 30$. Then

$$P = (1+0.035)^{-30} 24000 \approx 8550.67.$$

Example A debt of 3000, which is due six years from now, is instead to be paid off by three payments: 500 now, 1500 in three years, and a final payment at the end of five years. What should this payment be if an interest rate of 6 percent compounded annually is assumed?
Let a be the final payment due in five years. We shall set up an equation of value to represent the situation at the end of five years, for in that way the coefficient of a will be 1. Notice that at year 5 we compute the future value of 500 and 1500 and the present value of 3000. The equation of value is

$$(1.06)^5 500 + (1.06)^2 1500 + a = 3000(1.06)^{-1}.$$

Solving this equation for a gives $a \approx$ 475.7

When one is considering a choice of two investments, a comparison should be made of the value of each investment at a certain time, as the next example shows.

Example Suppose that you had the opportunity of investing 4000 in a business such that the value of the investment after five years would be 5300. On the other hand, you could instead put the 4000 in a saving account that pays 6 percent compounded semiannually. Which investment is better?

We consider the value of each investment at the end of the five years. At that time the business investment would have value of 5300, while the saving account would have a value $4000(1.03)^{10} \approx$ 5357.66. Clearly the better choice is putting the money in the saving accounts.

Example You have decided to invest the money in interest- earning deposits. You have determined that suitable deposits are available at your Bank paying 13.5% compounded semiannually, at a local Trust Company paying 14% and at your Credit union paying 13.25 compounded monthly. What institution offers the best rate of interest?

To make the rates comparable the effective rates of interest corresponding to the nominal annual rates should be determined.

1) Bank: $i = 0.0675,\ m = 2$. Then

$$p = (1+0.0675)^2 - 1 =).13956.$$

1) Trust company : $i = 0.14,\ m = 1$. Then

$$p = 0.14.$$

1) Credit union: $i = 0.01104,\ m = 12$. Then

$$p = (1.01104)^{12} - 1 = 0.14085.$$

Thus Credit union offers better conditions.

In case compounding is continuous, then an effective rate is calculated by equality: $r = \mathrm{e}^{i} - 1$.

There are legal the limits on the interest rate banks can offer for various types of accounts. In the last decade these limits have not compared favourably with interest rate that can be obtained from non-bank investments and, in an effort to attract more deposits, some banks have adopted not only the ultimate in compounding, continuous compounding, but also the *modified* year, 365/360 which is greater than one.

Example The writer has before him a newspaper advertisement that the effective rate on 8 percent is 8.45 percent.

To obtain this result, we replace the exponent in $e^{0.08} - 1$ by

$$\frac{365}{360} \cdot 0.08 \approx 0.0811,$$

to obtain

$$r = e^{0.0811} - 1 \approx 0.0845.$$

Example A bank states that the effective interest on savings accounts that earn continuous interest is 7 percent. Find the nominal rate.

Here,

$$r = 0,07, \quad j = \ln(1 + r) \approx 0.067, \quad or 6,7\%.$$

Self-control exercises

1. Determine the exact time for:

a) April 25 to October 14;

(b) July 30 to February 1.

Ans: (a) 172 days (b) 186 days

2. Compute the exact interest for:

(a) 1975.00 at 14.5% for 215 days;

(b) 844.65 at 13.25% from May 30 to January 4.

Ans: (a) 168.69 (b) 67.15

3. What principal will earn:

(a) 83.52 interest at 12% in 219 days?

(b) 34.40 interest at $9\frac{3}{4}$% from October 30, 1990 to June 1, 1991?

Ans: (a) 1160.00 (b) 601.77

4. Answer each of the following:

(a) What was the rate of interest, if the interest on a loan of 675 for 284 days was 39.39?

(b) How long will it take for 2075 to earn 124.29 interest at $8\frac{1}{4}$% p.a.? (State your own answer in days)

(c) If 680 is worth 698.70 after three months, what interest rate was charged?

(d) How many months will it take 750 to grow to 805 at 11% p.a.?

Ans: (a) 7.5% (b) 265 days (c) 11 (d) 8 months

5. Solve each of the following.

(a) What principal will have a maturity value of 665.60 at 10% in 146 days?

(b) What is the present value of 6300 due in 16 months at $7\frac{3}{4}$%

Ans: (a) 640.00 (b) 5709.97

6. What principal will earn 61.52 at 11.75% in 156 days?

Ans: 1225.03

7. What sum of money will earn 112.50 from September 1, 1992 to April 30, 1993 at 14.5%

Ans: 1175.06

8. At what rate of interest must a principal of 1435.00 be invested to earn interest of 67.57 in 125 days?

Ans: 13.75%

9. At what rate of interest will 1500.00 grow to 1622.21 from June 1 to December 1?

Ans: 16.25%

10. In how many months will 2500.00 earn 182.29 interest at 12.5%?

Ans: 7 months

11. In how many days will 3100.00 grow to 3426.39 at 15.75%?

Ans: 244 days

12. Compute the accumulated value of 4200.00 at 11.5% after eleven months?

Ans: 4642.75

13. What is the amount to which 1550.00 will grow from June 10 to December 15 at 14%?

Ans: 1661.77

14. What sum of money will accumulate to 1460.80 in eight months at 16%?

Ans: 1320.00

15. What principal will amount to 3441.62 if invested at 13% from November 1, 1991 to May 31, 1992?

Ans: 3200.00

16. What is the present value of 3780.00 due in nine months if interest is 12%?

Ans: 3467.89

17. Find the present value on June 1 of 1785.00 due on October 15 if interest is 15%.

Ans: 1690.52

18. Debt payments of 1750.00 and 1600.00 are due four months from now and nine months from now respectively. What single payment is required to pay off the debt today if interest is 13.5%?

Ans: 3127.53

19. A loan payment of 1450.00 was due 45 days ago and a payment of 1200.00 is due in 60 days. What single payment made 30 days from now is required to settle the two payments if interest is 16% and the agreed focal date is 30 days from now?

Ans: 2682.09

20. Debt obligations of 800.00 due two months ago and a payment of 1200.00 due in one month are to be repaid by a payment of 1000.00 today and the balance in three months. What is the size of the final payment if interest is 15.5% and the agreed focal date is one month from now?

Ans: 1044.38

21. An obligation of 10000.00 is to be repaid by equal payments due in 60 days, 120 days and 180 days respectively. What is the size of the equal payments if money is worth 13% and the agreed focal date is today?

Ans: 3474.83

22. Payments of 4000 each due in four, eight and twelve months respectively are to be settled by five equal payments due today, three months from now, six months from now, nine months from now and twelve months from now. What is the size of the equal payments if interest is 12.75% and the agreed focal date is today?

Ans: 2351.17

23. A loan of 5000.00 due in one year is to be repaid by three equal payments due today, six months from now and one year from now respectively. What is the size of the equal payments if interest is 14% and the agreed focal date is today?

Ans: 1559.86

24. Three debts, the first for 1000 due two months ago, the second for 1200 due in 2 months and the third for 1400 due in 4 months, are to be repaid by a single payment today. How much is the single payment if the money is worth 11.5% p.a. and the agreed focal date is today?

Ans: 3544.91

25. Debts of 700 due 3 months ago and of 1000 due today are to be paid by payment of 800 in two months and a final payment in five months. If 15% interest is allowed and the focal date is five months from now, what is the size of the final payment?

Ans: 1002.50

26. A loan of 3000 is to be repaid in three equal instalments due 90, 180 and 300 days respectively after the date of the loan. If the focal date is the date of the loan and interest is 10.9% p.a., find the size of the instalments.

Ans: 1056.12

27. Three debts, the first for 2000 due three months ago, the second for 1500 with interest of 12% due in nine months and the third for 1200 with interest of 15% due in eighteen months, are to be pain in three equal instalments due today, six months from now and one year from now respectively. If money is worth 13% and the agreed focal date is today, determine the size of the equal payments.

Ans: 1694.41

28. A debt of 5000 due in three years with interest at 10% is to be settled by four equal payments due today, one year, two years and three years from now respectively. Determine the size of the equal payments if money is worth 12% and the agreed focal day is today.

Ans: 1850.98

1.4 Compound interest

Consider the situation when the interest earned by an invested sum of money (or principal) is reinvested so that it too earns interest. That is, the interest is converted into principal and hence is "interest on interest."

Example Assume that the principal of 1000 is invested for two years at the rate 5% compounded annually. After the first year the sum of the principal plus interest is $(1+0.05)1000 = 1050$. At the end of the second year the value of the investment is $(1+0.05)1050 = 1102.5$. The last sum represents the original principal plus all accrued interest; it is called the *accumulated amount* or *compound amount*. In this case we say, that interest is capitalized. The difference between the compound amount ant the original principal is called the compound interest. In the above case the compound interest is 102.5.

Consider this situation generally. Suppose that a principal of P is invested at rate r compounded annually, then the amount after one year is

$$S_1 = P + rP = (1+r)P.$$

At the end of the second year the compound amount is

$$S_2 = (1+r)S_1 = (1+r)(1+r)P.$$

In general, the compound amount S of a principal P at the end of n years at the rate of r compounded annually is given by

$$S = (1+r)^n P. \tag{4}$$

Example Find the compound amount after ten years, if 10000 is invested at 4 percent compounded annually.

Applying formula (4) we obtain that

$$S = (1+0.04)^{10} 10000 \approx 14802.$$

The compound interest after ten years

$$I = S - P = 14802 - 10000 = 4802.$$

The quantities P, S, I in what follows is in some currency.

A time interval t we call *conversion period* if compound interest is computed at the end of each interval (compounding take place every period t.) The process, when the interest is capitalized we call *recalculation*.

Compound interest is usually computed periodically throughout the year. If compound interest is computed every month ($m = 12$) it is said to be compounded monthly. Each month is called a conversion period or interest period. If compound interest is computed every 3 months ($m = 4$) it is said to be compounded quarterly. If compound interest is compounded every 6 month ($m = 2$) it is said to be compounded semiannually. Interest may also be compounded annually, weakly, daily, etc. continuously. The annually interest rate is called *n*ominal rate if compound interest is compounded more when one time in year. The interest rate per conversion period is called *actual interest rate.*

In what follows we use such denotes:

P we denote original principal (present value);
$S-$ compound amount (future value or maturity value or accumulated value);
$I-$ amount of interest;
$i-$ actual interest rate;
$r-$ quoted interest rate (nominal rate).
$m-$ number of conversion periods per year;
$n-$ total number of conversion periods.

Example Suppose the principal of 1000 is invested for ten years, but the time compounding takes place every three months (quarterly) at the rate 1.5 percent per quarter. Then there are four interest periods or conversion periods per year, and in ten years there are $n = 10{\cdot}4 = 40$ conversion periods. Thus the compound amount with $r = 0.012$ is

$$S = (1 + 0.015)^{40}1000 \approx 1814.02.$$

Usually the interest rate per conversion period is stated as an annual rate. Here we will speak of an annual rate of 6 percent compounded quarterly so that the rate per conversion period (actual interest rate) $i = \frac{0.06}{4} = 0.015$.

In what follows unless otherwise stated, all interest rate will be assumed to be annual (nominal) rates. It is clear, that the nominal rate r annually does not necessarily mean that an investment increases in value by $100r\%$ in a year's time.

The formula

$$S = (1 + i)^n P$$

gives the compound amount S of a principal P at the end of n conversion periods at the rate of r per conversion period

$$n = mt, \;\; i = \frac{r}{m}, \; t - \text{duration in years}$$

.

Example The sum of 3000 is placed in a saving account. If the money is worth 6 percent compounded semiannually, what is the balance in the account after seven years?

We have $P = 3000$ and $m = 2$ thus $n = 7 \cdot 2 = 14$. The rate per conversion period is $\frac{0.06}{2} = 0.03$. We have

$$S = (1 + 0.03)^{14}3000 \approx 4537.$$

Example How long will it take 600 to amount 900 at an annual rate of 8 percent compounded quarterly?

It is clear that $m = 4$ and $i = \frac{0.08}{4}$. n will be the number of conversion periods it takes for a principal of $P = 600$ to amount to $S = 900$. Applying equality (4) we obtain

$$900 = 1.02^n 600.$$

Thus $1.02^n = 1.5$. Taking the natural logarithms of both sides, we have

$$n = \frac{\ln 1.5}{\ln 1.02} \approx \frac{0.40547}{0.01980} \approx 20.478.$$

The number of years that corresponds to 20.478 quarterly conversion periods is $20.478/4 = 5.1195$, which slightly more than 5 years and 1 month.

If a sum of money is invested at compound interest, its value increases exponentially with time. This can be shown by beginning with the formula for compound amount

$$S = (1+i)^n P.$$

and replacing n with the equivalent expression mt. This result is

$$S = (1+i)^{mt} P$$

where m is an integer. Applying the law of exponents, the equation becomes

$$S = \big((1+i)\big)^{mt} P.$$

Thus, $S = S(t)$. Given values for P, i and m the function ia an exponential equation of the form $y = ab^x,\ B > 1$.

1.5 Compound discount

In compound interest problems, the compound interest rate r is applied to principal P. Sometimes a loan is transacted by *discount note*. In such a case, the cost of borrowing is called the *discount*, D. The discount is computed as a percentage of the maturity value S. This percentage as in case of simple interest is called the *discount rate* d.

Thus if length of time is (in one year), then discount (cost of borrowing) are such: $D = Sd$, or

$$d = \frac{S-P}{S},$$

here S is maturity after one year. Therefore

$$P = (1-d)S.$$

The discount rate d can be expressed by interest rate per converse period:

$$d = \frac{i}{1+i}.$$

The process, when we find present value B in arbitrary time moment before maturity S, in case compound interest is called *discounting*.

Suppose, that actual interest rate is i, number of conversion periods is m and time interval between present value and maturity is t (in year). Then present value can be obtained by formula

$$B = \frac{S}{(1+i)^{mt}} =: \nu^{mt} S.$$

The factor

$$\nu = \frac{1}{(1+i)}$$

is called *compound discount factor*.

We note, that in the special literature amount $B-P$ is called the *discount* and discounted value $B(t)-$ proceed. Thus

$$D = B - P = (1-\nu^n)S.$$

1.6 Equation of values

Sums of money have different values at different points in intervals. And is clear that is the reason that that amounts of money at different points in time are not comparable. To make such sums of money comparable we introduce notation of focal date (comparison date), which must be selected and allowance must be done for interest form due dates of the sums of money to the selected point in time.

Using the same argument which was made with simple interest we consider two positions of the due dates relative to the focal date:

1) The due date falls before the focal date. In this case we shall use amount formula $S = (1+i)^n P = (1+i)^{mt}P$;

2) The due date falls after the focal date. In this case we shall use present value formula

$$P = (1+i)^{-n}S(1+i)^{-mt}P.$$

Example 400000 is payable three years from now. Suppose the money is worth 14% compounded semi-annually. Determine the equivalent value 1) seven years from now; 2) now.

In this case we have, that the due date is before the focal date. In this case we use amount formula. Thus we have

$P = 400000,\quad i = 0.07,\quad n = 8.$

Then

$$S = (1+0.07)^8 \cdot 400000 = \ 687274.$$

For the second case we have that the due date is after the focal date, therefore

$S = 400000,\quad i = 0.07,\quad n = 6.$

Then

$$P = (1+0.07)^{-6} \cdot 400000 = \ 266537.$$

Consider more complicated examples.

Example Debt payment of 40000 due five months ago, 60000 due today and 80000 due in nine months are to be combined into one payment due three months from today at 15% compounded monthly.

Set the equivalent values to focal date by S_1, S_2 and P_1, here

$S_1 = (1.0125)^8 40000 = \ 44179;$

$S_2 = (1.0125)^3 60000 = \ 62278;$

$P_1 = (1.0125)^{-6} 40000 = \ 74254.$

Sum of all three amounts represent the equivalent single payment to settle the debt three month from now. It is 180711.

Example Payments of 10000 are due at the end of each of the next five quarters. Determine the equivalent single payment which would settle the debt payments now if the interest is 15% compounded quarterly.

An equivalent single payment we denote by P and the date payment by

$$P_1, P_2, P_3, P_4, P_5.$$

Then

$$P = 10000(1.0375)^{-1} + 10000(1.0375)^{-2} + 10000(1.0375)^{-3} +$$
$$10000(1.0375)^{-4} + 10000(1.0375)^{-5} = 44832.5.$$

Now we consider the problem of finding the value of two or more equivalent payments.

Example Debt payments of 100000 due today and 200000 due one year from now are settled by a payment of 150000 three months from now and a final payment eighteen months from now. Determine the size of the final payment if interest rate is 18% compounded quarterly.

Two payments are done before the focal date and the first replacement payment is due before the focal date too. Therefore we apply amount formula for the future values. We have

$$S_1 + S_2 = S_3 + x; \quad \text{here} \quad S_1 = (1.045)^6 100000 = 130000;$$

$$S_2 = (1.045)^2 200000 = 218000; \ S_3 = (1.045)^5 150000 = 186900.$$

Solving this equation we obtain the final payment 161104.

Example What is the size of the equal payments which must be made at the and of each of the five quarters to settle a debt of 300000 due now if the money is worth 16% compound quarterly?

We assume that the focal date is now. The equal payments we denote by x. Then

$$300000 = x(1.04)^{-1} + x(1.04)^{-2} + x(1.04)^{-3} + x(1.04)^{-4} + x(1.04)^{-5}.$$

Solving this equation we obtain, that $x = 67388$.

Definition The date on which a single amount of money is equal to the sum of dated sums of money we call the *equate date.*

To find the equate date we need to find the time moment n.

Example A financial liability required the payments of 200000 in six months, 300000 in fifteen months and 500000 in 24 months. When can all these payments discharged by the single payment 100000 equal to the sum of required payments if money is worth 18% compounded monthly?

We select a focal date starting at now. We have that $i = 0.0125$. Then the equation of the values yield:

$$100000 = 200000(1.0125)^{-6} + 300000(1.0125)^{-15} + 500000(1.0125)^{-24}.$$

Summing right hand side we obtain

$$(1.0125)^{-n} = 0.805732, \quad \text{or} \quad -n \ln(1.0125) = \ln 0.805732.$$

Then $n = 17.388$. Therefore the equated date is 17.4 months, or 529 days.

1.7 Compound amount when time is fractional. Continuous compounding

The main formula of the finding compound amount based on the compounding factor $(1+i)^n$, where n (time) is integer number or $(1+i)^{mt}$,. Now we consider this compound factor with rational time variable. Using the formula with n as a fractional number is the theoretical correct method because in this case we obtain exact accumulated value. Consider example

Example Find the accumulated value of 100000 invested for two years and nine months at 15% compounded annually using the exact method. We have that

$P = 100000; \quad i = 0.15, \quad n = 2.75$. Then

$$S = (1.15)^{2.75}100000 = 146865.$$

Example Find the accumulated value of 350000 invested in August 31 2001 until June 30, 2004 at 15% compounded quarterly using the exact method. We have that

$P = 350000; \quad i = 0.0.0375$. The time interval indicated in the example contains 2 years and 10 months. Then the number of quarters $\quad n = 11,(3)$. Then

$$S = (1.0375)^{11.3333}3500000 = 531210.$$

Example Suppose we signed a promissory note at the bank 300000 due in 27 months. If a bank charges interest at 16% compounded semi-annually, determine the proceeds of the note.

The maturity value of the amount is $S = 300000, \quad i = 0.08, \quad n = \frac{27}{12} \cdot 2 = 4.5$. Then the proceeds of the note is such:

$$P = \frac{S}{1.08^{4.5}} = 212185.$$

We have considered the example where an interest has been compounded annually, semi-annually, monthly. Interest may also be compounded weekly, daily, hourly and etc. In what follows we will employ a 365-day year.

Example A person invested for 3 years 1800 at 10 percent compounded daily. Find the compound amount.

We have

$$S = (1+i)^n P = 18000(1 + \frac{0.1}{365})^{1095}.$$

Thus the compound amount is

$$S = 18000 \cdot 1.3498025 = 24296.45.$$

The previous example has involved daily compounding of interest. We may go further and compound every minute, every second, etc. As the number of compounding per year, m increases without bound, the interest had to be $c$ompounded continuously.

To determine the formula for the future value of an amount P when it is compounded continuously, we begin with

$$S = P\big(1 + \frac{r}{m}\big)^{mt}$$

where r is the nominal rate, m is the number of conversion periods per year, and t is the number of years.

Suppose that m increases without bound. Then the preceding formula for S is rewritten as

$$S = \lim_{m\to\infty}\left(\left(1+\frac{1}{\frac{m}{r}}\right)^{\frac{m}{r}}\right)^{tr}.$$

Hence, as m increases, then m/r increases too. In the first section, considering sequences, we have proved that

$$\lim_{m\to\infty}\left(\left(1+\frac{1}{\frac{m}{r}}\right)^{\frac{m}{r}}\right)^{tr} = \mathrm{e}^{rt}.$$

Thus, when interest is compounded continuously at a nominal rate r the future value S given by:

$$S = P\mathrm{e}^{rt}.$$

The formula for present value at continuous compounding is found by beginning with the formula for compound amount $S = P\mathrm{e}^{rt}$. Solving it for P we have

$$P = S\mathrm{e}^{-rt}.$$

Example How long does it take money to double itself at 10 percent compounded continuously.

Beginning with the compound amount formula for S we have

$$S = P\mathrm{e}^{0.1t},$$

where t is the number of years. Since P is to double, we replacing S with $2P$ obtain

$$2P = P\mathrm{e}^{0.1t}.$$

Dividing both sides by P and restarting result in logarithmic form we get

$$0.1t = \ln 2.$$

Solving for t yields $t \approx 6.9315$.

Thus, it takes approximately 6.9 years for money to double itself at 10 percent compounded continuously.

Example What sum of money should be invested for 5 years at 6 percent compounded continuously in order to provide a compound amount of 9000?

In this case, we have

$$P = S\mathrm{e}^{-rt} = 9000\mathrm{e}^{-0.06\cdot 5} \approx 6667.$$

More detail consider relation

$$S = P\mathrm{e}^{rt}.$$

The nominal discrete rate r in this relation we change to δ :

$$S = P\mathrm{e}^{\delta t}.$$

The parameter δ is called *force of interest* and can hold it continuous parameter.

In the case $\delta = \delta(t)$, future value and principal value can be found by relations:

$$S = \mathrm{e}^{\int\limits_0^n \delta(t)\mathrm{d}t} P, \quad P = \mathrm{e}^{-\int\limits_0^n \delta(t)\mathrm{d}t} .S$$

Suppose that force of interest is linear $\delta(t) = \delta + kt$ where δ is initial value of force of interest, and a is change of force of interest in time unit. Then applying formula given above we deduce

$$S = \mathrm{e}^{\int\limits_0^n \delta + at\mathrm{d}t} P = \mathrm{e}^{\delta n + \frac{an^2}{2}} P.$$

1.8 Equivalence of the rates

In the process of storage or discount various rates can be used. In this section we consider problem, when change rates give the same result.

Denote i_s, d_s a simple rate and simple discount rate respectively and i, d a compounded rate and compounded discount rate, respectively, for some converse period and $j-$ nominal rate.

Consider problem equivalence of the simple and compounded rates. We have

$$1 + ni_s = (1 + i)^n.$$

From it follows the following equalities of equivalents of rates:

$$i_s = \frac{(1+i)^n}{n} - 1, \quad i = \sqrt[n]{1 + ni} - 1.$$

Find equivalence between the magnitudes i, j, d. We have that

$$1 + i = (1 + \frac{j}{m})^m.$$

Thus

$$i = (1 + \frac{j}{m})^m - 1, \quad j = m(\sqrt[m]{1+i} - 1).$$

For i and d the have the following relations:

$$i = \frac{d}{1-d}, \quad d = \frac{i}{1+i}.$$

Consider problem equivalence between the discount rate and simple rates, when time bases are $K = 360$ or $K = 365$. Let n converse period in year. Then we have

$$i_s = \frac{d_s}{1 - nd_s}, \quad d_s = \frac{i_s}{1 + ni_s}.$$

Consider the last equalities, when converse period is daily. Then $n = \frac{t}{K}$, where $t-$ number of days, K is time base.

In the case of simple interest we have

$$i_s = \frac{d_s}{1 - td_s}, \quad d_s = \frac{i_s}{1 + ti_s},$$

here $t = \frac{n}{K}$, $n-$ number of the days, K time base. in case $K = 360$, we have

$$i_s = \frac{360 d_s}{360 - n d_s}, \quad d_s = \frac{360 i_s}{360 + n i_s}.$$

Consider the generale case. Let i_1, d_1 be simple interest and discount rates in bases K_1 respectively and i_2, d_2 be interest and discount rates in bases K_2. Find relationships between these magnitudes.

It's easy to see that

$$1 + \frac{n}{K_1} i_1 = 1 + \frac{n}{K_2} i_2 \Rightarrow \qquad i_1 = \frac{K_1}{K_2} \cdot i_2.$$

Similarly

$$d_1 = \frac{K_1}{K_2} d_2,$$

$K_1 = 360, \quad K_2 = 365.$

Using the relation above we deduce that

$$i_1 = \frac{d_1}{1 - t d_1} = \frac{\frac{K_1}{K_2} d_2}{1 - \frac{n}{K_1} \frac{K_1}{K_2} d_2} = \frac{K_1 d_2}{K_2 - n d_2}$$

or

$$i_2 = \frac{K_2 d_1}{K_1 - n d_1}.$$

The same arguments yields

$$d_1 = \frac{K_1 i_2}{K_2 + n i_2}.$$

Example Find discount rate in base $K = 360$, for the day number $n = 255$ which be equivalent to 40% simple interest rate in base $K = 365$.

$$d_s = \frac{360 \cdot 0.4}{365 + 255 \cdot 0.4} \approx 0.3.$$

$$i_s = \frac{360 d_s}{360 - t d_s}, \quad d_s = \frac{360 i_s}{360 + t i_s}.$$

Suppose that rates period are daily, when time base is $K = 365$ and discount rate base is $K = 360$, then we have

$$i_s = \frac{365 d_s}{360 - t d_s}, \quad d_s = \frac{360 i_s}{365 + t i_s}.$$

Example Find discount rate when day base $K = 360$, for time interval $t = 255$ days, which would be equivalent for the simple rate 40% with day base $K = 365$.

We have that

$$d_s = \frac{360 \cdot 0.4}{360 + 255 \cdot 0.4} \approx 0.3.$$

Consider equivalence among simple and compounded rates.

For the rates i_s and i we have

$$i_s = \frac{(1+\frac{j}{m})^{mn} - 1}{n}, \quad j = m(\sqrt[mn]{1+ni_s} - 1).$$

Further, for d_s and i we have

$$d_s = \frac{1-(1+i)^n}{n}, \quad i = \sqrt[n]{1-nd_s} - 1).$$

Equivalence for the d_s and j :

$$d_s = \frac{1-(1+\frac{j}{m})^{mn}}{n}, \quad j = m(\sqrt[mn]{1-di_s} - 1).$$

We find equivalence among nominal rate and force of rate. We have

$$(1+i)^n = \mathrm{e}^{\delta n}.$$

Then

$$i = \mathrm{e}^{\delta} - 1.$$

If rate r is nominal and m is number of converse periods, then from equality

$$(1+\frac{r}{m})^m = \mathrm{e}^{\delta}$$

we deduce that

$$\delta = m \cdot \ln(1+\frac{r}{m}).$$

Equivalence among δ and d follows from the relation:

$$(1-d)^{-1} = \mathrm{e}^{\delta}.$$

Thus,

$$\delta = -\ln(1-d), \quad d = 1 - e^{-\delta}.$$

1.9 Effective rate

Consider the following situation. If 1 is invested at a nominal rate of 8 percent compounded quarterly for one year, then the dollar will earn 8 percent more that year. The compounded interest is

$$S - P = (1.02)^4 - 1 \approx 0.082432$$

which about 8.24 percent of the original dollar. That is 8.24 percent is the rate of interest compounded annually that is actually obtained, and it is called effective rate.

Definition The percentage $p100\%$ compounded annually which is equivalent to $i100\%$ compounded in m times a year we call effective annual interest rate.

This interest rate p can be computed by ratio

$$p = \frac{(1+i)^m P - P}{P} = (1+i)^m - 1.$$

Thus the effective annual interest rate $p = (1+i)^m - 1$, where m number of conversion periods for one year and i is interest rate per conversion period.

Definition about effective rate yields that:

1) The nominal annual rate is the effective rate of interest if it is compounded annually.

2) The effective rate of interest increases as the number of conversion periods of the given nominal annual rate per year increases.

Example Suppose that money is invested at 8 percent compounded quarterly, then $m = 4$, $r = 0.08$ and $i = \frac{0.08}{4} = 0.02$. then

$$p = (1+i)^m - 1 = (1.002)^4 - 1 = 0.0824.$$

Thus, 8 percent compounded quarterly is equivalent to 8.24 percent compounded annually.

Example How many years will it take for a principal P to double at the effective rate of i.

Let n be the number of years it takes. When P doubles then the compound amount S is $2P$. Thus $2P = (1+i)^n P$. So

$$2 = (1+i)^n. \text{ Thus, } \ln 2 = n \ln(1+i), \; n = \frac{\ln 2}{\ln(1+i)} \approx \frac{0.69315}{\ln(1+i)}.$$

Choosing $i = 0.06$ we obtain that $n \approx 11.9$.

Self-control exercises

1. What is accumulated value of 500.00 in fifteen years at 13.5% compounded:

(a) annually?

(b) quarterly?

(c) monthly?

Ans: (a) 3341.24 (b) 3663.54 (c) 3745.47

2. What is the amount of 10000.00 at 16.5% compounded monthly:

(a) in four years?

(b) in eight and a half years?

(c) in twenty years

Ans: (a) 19261.12 (b) 40266.93 (c) 265099.04

3. Landmark Trust offers 5-year investment certificates at 15% compounded semi-annually.

(a) What is the value of 2000

(b) How much of the maturity value is interest?

Ans: (a) 4122.06 (b) 2122.06

4. Western Savings offers three - year term deposits at 15.25% compounded annually while your Credit Union offers such deposits at 14.5% compounded quarterly. If you have 5000 to invest what is the maturity value of your deposit

(a) at Western Savings?

(b) at your Credit Union?

Ans: (a) 7654.08 (b) 7665.57

5. Find the compound amount and the compound interest of

(a) 1800.00 invested at 14% compounded quarterly fo 15.5 years;

(b) 1250.00 invested at 13.5% compounded monthly for 15 years.

Ans: (a) 15 190.57; 13390.57 (b) 9363.67 ; 8113.67

6. The Peel Company borrowed 20000.00 at 10% compounded semi-annually and made payments toward the loan of 8000.00 after two years and 10000.00 after three and a half years. How much is required to pay the loan off one year after the second payment?

Ans: 9791.31

7. A.B. deposited 1750.00 in an SEB on March 1, 1996 at 10% compounded quarterly. Subsequently the interest rate was changed to 12% compounded monthly on September 1, 1998 and to 11% compounded semi-anually on June 1, 2000. What will the value of the SEB deposit be on December 1, 2006 if no further changes in interest are made?

Ans: 5537.42

8. An investment of 2500 is accumulated at 14% compounded quarterly for two and a half year. At that time the interest rate is changed to 13.5% compounded monthly. How much is the amount of the investment two years after the change in interest rate?

Ans: 4612.63

9. To assure that funds are available to repay the principal at maturity a borrower deposits 2000 each year for three years. If interest is 13% compounded quarterly, how much will the borrower have on deposit four years after the first deposit was made?

Ans: 8855.18

10. A.B. started a registered retirement savings plan on February 1, 1984 with a deposit of 2500. She added 2000 on February 1, 1985 and 1500 on February 1, 1986. What is accumulated value of her bank account on August 1, 1994 if interest is 14% compounded quarterly?

Ans: 22826.47

11. Find the present value and the compound discount of 4000 due in seven years and six months if interest is 14.8% compounded quarterly?

Ans: 1344.92; 2655.08

12. Find the principal which will accumulate to 6000 in fifteen years at 15% compounded monthly.

Ans: 641.28

13. A sum of money has a value of 3000 eighteen months from now. If money is worth 18% compounded monthly what is its equivalent value?

(a) now?

(b) one year from now?

(c) three years from now?

Ans: (a) 2294.73 (b) 2743.63 (c) 3922.02

14. Payments of 1000, 1200 and 1500 are due in six months, eighteen months and thirty months from now respectively. What is the equivalent single payment two years from now if money is worth 16% compounded quarterly?

Ans: 3950.07

15. An obligation of 10000 is due one year from now with interest at 13% compounded semi-annually. The obligation is to be settled by a payment of 6000 in six months and a final payment in fifteen months. What is the size of the second payment if interest is 18% compounded monthly?

Ans: 3459.54

16. A.B. owes 3000 due in two years with interest at 16% compounded semi-annually and 2500 due in fifteen months at 13% compounded quarterly. If Joe wants to discharge these debts by making two equal payments, the first one now and the second eighteen months from now, what is the size of the two payments if money is worth 15% compounded monthly?

Ans: 2381.18

17. Debt payments of 400.00 due today, 500.00 due in eighteen months and 900.00 due in three years are to be combined into a single payment due two years from now. What is the size of the single payment if interest is 16% p.a. compounded quarterly?

Ans: 1857.55

18. Debt payments of 2600.00 due one year ago and 2400.00 due two years from now are to be replaced by two equal payments due one year from now and four years from now respectively. What is the size of the equal payments if money is worth 13.5% p.a. compounded semi-annually?

Ans: 3271.60

Homework exercises

1) Simple interest

1. Compute the interest on 5000 at 8 percent for 9 months.

2. Find the interest rate if 1000 earns 45 interest in 4 months.

3. Find the amount if 20000 is borrowed at 6 percent for 3 months.

4. A credit card holder has owed the credit card company 2000 for a month and receives a bill containing an interest charge of 30. Find the interest rate.

5. How much must be deposited in an account paying 7 percent if interest of 1000 is to be earned in 24 months.

6. How many months will it take at 8 percent interest for 2000 to grow to an amount of 2400?

7. A.B deposits 10000 in an employee's savings account at 6 percent. How many months will it be until the amount in the account is 11000.

8. Find the present value of 12000 receivable 18 months from now if the interest rate is 8 percent.

9. How much will A.B. have to invest now in an employees savings account at 7 percent in order to have 10000 in the account 18 months from now?

10. Find the proceeds of a 20000, 18 month loan from a bank if the discount rate is 12 percent.

11. Find discount rate if interest rate is 16 percent.

12. A.B. wants 2000 now from a bank, to be repaid 18 months from now. How much will the repayment be if the discount rate is 15 percent.

13. At what rate of interest must a principal of 14000 be invested to earn interest of 6120 in 150 days?

14. At what rate of interest will 15000 grow to 15820 from June 1 to December 15?

15. In how many days will 42000 grow to 42600 at 10.75%?

16. A.B. borrows 2000 for 10 years, interest rate is 6 percent. After 4 years he returns 70 percent of the loans. After 8 years he returns 60 percent of the borrowed amount. How much will the bank earns in 10 years?

17. Three debts, the first for 3000 due two months ago, the second for 2000 due in 5 months and the third for 4000 due in 8 months, are to be repaid by a single payment today. How much is the single payment if the money is worth 10% p.a. and the agreed focal date is

a) today?

b) 4 month from now.

18. A loan of 8000 is to be repaid in three equal instalments due 120, 220 and 320 days respectively after the date of the loan. If the focal date is 100 after now and interest is 12% , find the size of the instalments.

19. A loan of 10000 is to be repaid in two equal instalments due 200 and 360 days respectively after the date of the loan. Find the focal date (if it is possible) interest rate is 12% , and the instalments are equals to 5400.

2) Compound interest rate

1. Find the present value and the compound discount of

(a) 3600.00 due in 9 years if interest is 15% compounded semi-annually;

(b) 9000.00 due in 5 years if money is worth 16.8% compounded quarterly.

2. What is accumulated value of 35000 in ten years at 15% compounded:

(a) annually?

(b) quarterly?

(c) monthly?

3. Bank offers five - year term deposits at 16% compounded annually while your Credit Union offers such deposits at 15% compounded quarterly. If you have 6000 to invest what is the maturity value of your deposit

(a) at Bank?

(b) at your Credit Union?

4. Find the present value and the compound discount of

(a) Lt4500 due in 7.5 years if interest is 12% compounded semi-annually'

(b) Lt8000 due in 6 years if money is worth 18% compounded quarterly.

5. An investment of 5000 is accumulated at 12% compounded quarterly for two and a half year. At that time the interest rate is changed to 15% compounded monthly. How much is the amount of the investment two years after the change in interest rate?

6. A demand loan of Lt9000 is repaid by payments of Lt4000 after fifteen months, Lt6000 after thirty months and a final payment after four years. If interest was 13% for the first two years and 12% for the remaining time period and compounding is quarterly, what is the size of final payment?

7. Payments of 4000, 8000 and 9000 are due in ten months, eighteen months and thirty months from now respectively. What is the equivalent single payment two years from now if money is worth 12% compounded quarterly?

8. Debt payments of 4000 due today, 5000 due in eighteen months and 8000 due in three years are to be combined into a single payment of 21200. What is the time moment, respect to today, when this single paymant is due if interest is 12% compounded monthly?

9. Payments of 6000, 8000 and 6000 are due in ten months, eighteen months and thirty months from now respectively. The single payment of 21500 is due 14 months from now. Find the worth of money if it is known that money are compounded quarterly?

10. What sum of money deposited now at 8% compounded quarterly will provide just enough money to pay a 2000 debt due seven years from now?

11. Find the effective rate of 24 percent compounded monthly.

12. Find the effective rate of 9 percent compounded daily (365 days per year).

13. Find nominal interest rate which is equivalent to the 18% effective rate.

14. Find discount rate when day base $K = 340$, for time interval $t = 255$ days, which would be equivalent for the simple rate 40% with day base $K = 365$.

15. Find interest rate when day base $K = 300$, which would be equivalent for the simple rate of 35% with day base $K = 360$.

16. Find interest rate when day base $K = 365$, for time interval $t = 320$ days, which would be equivalent for the simple discount 20% with day base $K = 360$.

17. Find simple discount rate when day base $K = 365$, which would be equivalent for the simple discount 20% with day base $K = 300$.

18. Find simple interest rate which be equivalent for the interest rate compounded monthly in 1) two years time interval; 1) five years time interval.

19. Find simple discount rate which be equivalent for the discount rate compounded monthly in 1) two years time interval; 1) five years time interval.