Algebraic integers, symmetric integer matrices and graphs

Chris Smyth, University of Edinburgh

Algebraic integers are roots of polynomials with integer coefficients, the term of highest degree having coefficient 1. The eigenvalues of integer matrices are algebraic integers, so perhaps integer matrices might be useful in the study of algebraic integers. Pisot and Salem numbers are special classes of algebraic integers. The aim of these lectures is to describe joint work with James McKee where we apply results about integer matrices to try to get an insight into these special algebraic integers. Symmetric integer matrices with entries in $\{-1,0,1\}$ can be visualised as kinds of graphs, and this enables us to give pictorial representations of most small Pisot and small known Salem numbers. We show how a restricted version of Boyd's conjecture can be proved, for Pisot and Salem numbers associated to graphs.

A tentative draft outline of the lectures is as follows.

Lecture 1 Symmetric matrices and graphs: eigenvalues, interlacing, effect of graph perturbation, cyclotomic graphs.

Lecture 2 Pisot and Salem numbers: Introduction, basic properties, Salem's construction of Salem numbers, Boyd's Conjecture on the closure of the set of all Pisot and Salem numbers.

Lecture 3 Constructing Salem numbers from trees, and from more general graphs. Pisot graphs from Salem graphs. Small Salem and Pisot numbers from graphs.

Lecture 4 Cyclotomic matrices and charged signed graphs.

Lecture 5 Proof of Boyd's Conjecture for Pisot and Salem numbers coming from a certain class of symmetric integer matrices.