ON THE RATE OF NORMAL APPROXIMATION IN $D[0,1]$

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Abstract : The paper gives estimates of the accuracy of the approximation of the probability distribution function of $\sup_{t\in[0,1]} |n^{-1/2}S_n(t)|$ for sums of random processes $S_n(t) = X_1(t) + \ldots + X_n(t)$ satisfying the the central limit theorem in the Skorokhod space $D[0,1]$.

1. Introduction

Let $X = \{X(t),\ t \in [0,1]\}$ be a centered stochastically continuous random process with cadlag (right continuous and having left limits) sample paths. Let $D[0,1]$ denotes the Skorohod space of cadlag functions on $[0,1]$ endowed with the usual topology. The process X is said to satisfy the central limit theorem (CLT) in $D[0,1]$ if $\mathcal{L}(n^{-1/2}(X_1 + \cdots + X_n))$ converges weakly to a Gaussian measure on $D[0,1]$. Here $X_1, X_2, ...$ are independent copies of X. The aim of the paper is to estimate the rate of convergence in the following limit theorem proved recently by Bloznelis and Paulauskas (1993 b) and Fernique (1993).

Theorem 1. Let $p, q \geq 2$. Let f, g be nonnegative increasing functions on $[0, +\infty)$. Let X be a r. process with mean 0, finite second moments, and sample paths in $D[0,1]$ satisfying

$$\forall s \leq t \leq u \qquad \mathbf{E}\big(|X(s) - X(t)| \wedge |X(t) - X(u)|\big)^p \leq f(u - s), \tag{1.1}$$

$$\forall s \leq t \qquad \mathbf{E}|X(s) - X(t)|^q \leq g(t - s) \tag{1.2}$$

and

$$\int_0 f^{1/p}(u) \cdot u^{-1-1/p} du < \infty, \quad \int_0 g^{1/q}(u) \cdot u^{-1-1/(2q)} du < \infty \qquad (1.3).$$

Then the process X satisfies the CLT in $D[0,1]$.

Denote $S_n = n^{-1/2}(X_1 + \cdots + X_n)$ and let $Y = \{Y(t),\ t \in [0,1]\}$ denotes the limiting Gaussian process, i.e., $\mathcal{L}(S_n)$ converges weakly to $\mathcal{L}(Y)$. For $x \in D[0,1]$

denote $\|x\| = \sup_{t\in[0,1]} |x(t)|$. In the paper we estimate the uniform (Kolmogorov) distance between the distributions $\mathcal{L}(\|S_n\|)$ and $\mathcal{L}(\|Y\|)$.

Theorem 2. Let $p, q \geq 2$. Let X be a centered stochastically continuous cadlag process. Assume for some $\alpha > 0$ the process X satisfies conditions (1.1) and (1.2) with the functions $f(u) = c \cdot u^{1+\alpha}$ and $g(u) = c \cdot u^{1/2+\alpha}$ respectively. Then there exists a constant $C_1 = C_1(Y, p, q, \alpha)$ such that for each $r \geq 0$

$$|\mathbf{P}(\|S_n\| \leq r) - \mathbf{P}(\|Y\| \leq r)| \leq C_1(1 + \mathbf{E}\|X\|^3)n^{-1/6}\log^{2/3}(n). \tag{1.4}$$

Here and in what follows letters C and c with indices or without denote absolute constants which may depend only on the quantities indicated in the brackets.

Theorem 2 is a particular case of more general results formulated in section 2 below. Denote

$$\mathbf{\Delta}(r, S_n, Y) = |\mathbf{P}(\|S_n\| \leq r) - \mathbf{P}(\|Y\| \leq r)|.$$

Theorem 2 gives the bound for $\mathbf{\Delta}(r, S_n, Y)$ which is uniform with respect to $r \geq 0$. Next theorem gives the so called non – uniform bound.

Theorem 3. Let $p > 3$ and $q \geq 2$. Let X be a centered stochastically continuous cadlag process. Assume for some $\alpha > 0$ the process X satisfies conditions (1.1) and (1.2) with the functions $f(u) = cu^{1+\alpha}$, $g(u) = cu^{1/2+\alpha}$ and $\mathbf{E}\|X\|^\beta < \infty$ for some $\beta > 3$. Then there exists a constant $C_2 = C_2(Y, p, q, \alpha, \beta)$ such that for each $r \geq 0$

$$|\mathbf{P}(\|S_n\| \leq r) - \mathbf{P}(\|Y\| \leq r)|(1 + r^3) \leq C_2(1 + \mathbf{E}\|X\|^3)n^{-1/6}\log^2(n). \tag{1.5}$$

Remarks. 1. Let F and G be increasing continuous functions defined on $[0, 1]$. The statement of theorems $1, 2$ and 3 remain true if one replaces the differences $|u - s|$ and $|t - s|$ in the right hand sides of (1.1) and (1.2) by $|F(u) - F(s)|$ and $|G(t) - G(s)|$ respectively.

2. The rate of order $n^{-1/6}$ appears usually when one consider the CLT for a Banach function space valued random elements, see, e.g., Paulauskas and Račkauskas (1989) and Bentkus et al (1990), moreover this rate is optimal in a sense.

Earlier, the rate of convergence in the CLT in $D[0,1]$ was considered by Paulauskas and Juknevičienė (1989) and Paulauskas and Stieve (1990). Results of the present paper (theorems 2, 3, 4, and 5) are simpler and sharper. The rate of convergence is obtained under considerably weaker conditions than those used in Paulauskas and Juknevičienė (1989) and Paulauskas and Stieve (1990).

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2. Main Results

Here we formulate and prove main results. In proofs one combines the finitedimensional approximation with the estimates of the rate of convergence in the CLT for l_∞^N valued random vectors. Such estimates of order $n^{-1/6}$ (here n denotes the number of random vectors in the summ) which are non sensitive with respect to the growing dimension ($N \to \infty$) were obtained in Bentkus (1982) and Račkauskas (1984).

Theorem 4. Assume the process X satisfies conditions of Theorem 1 with the parameters $p = 2, q \geq 2$. Then there exists a constant $C_3 = C_3(Y, q)$ such that for any sequence of entire numbers $\{N_n\, n \geq 1\}$, satisfying $N_n \geq n^{1/3},\ n \geq 1$ the following bound holds

$$\sup_{r \geq 0} \mathbf{\Delta}(r, S_n, Y) \leq$$

$$C_3\big((1 + \mathbf{E}\|X\|^3)n^{-1/6}\log^{2/3} n + [T_1(N_n^{-1})]^{1/2} + [N_n^{-1}\log^{1/2}\log(N_n)]^{1/2}\big), \quad (2.1)$$

where

$$T_1(s) = \int_0^s u^{-3/2} f^{1/2}(u)du + \int_0^s u^{-1-1/(2q)} g^{1/q}(u) \log^{1/2-1/(2q)}(1 + 1/u)du.$$

The main goal of theorem 4 is that the integrand of the first integral is precisely the same as that in condition (1.3), when $p = 2$. The logarithmic factor that appears in the second integral is, likely, superfluous.

The following theorem 5 gives the bound for the rate of convergence in the CLT due to Bézandry and Fernique (1992).

Theorem 5. Let $X, X_1, X_2, X_3, ...$ be i.i.d. stochastically continuous centered cadlag processes defined on the probability space $(\Omega_1, \mathcal{F}_1, P_1)$. Assume there exist increasing functions $f, g, \theta, f(0) = g(0) = \theta(0) = 0,\ \theta$ is concave, such that

$$\forall s \le t \le u,\ \ \forall A \in \mathcal{F}_1\ \ \mathbf{E}|X(s)-X(t)|^2 \wedge |X(t)-X(u)|^2 \mathbb{I}_A \le f(u-s)\theta(P(A)), \quad (2.2)$$

$$\forall s \le t \qquad \mathbf{E}|X(s) - X(t)|^q \le g(t-s). \quad (2.3)$$

Then there exists a constant $C_4 = C_4(Y, \theta, q)$ such that for any sequence of entire numbers $\{N_n\, n \ge 1\}$, satisfying $N_n \ge n^{1/3},\ n \ge 1$ the bound (2.1) holds with the constant C_4 instead of C_3 and with $T_1(\cdot)$ replaced by $T_2(\cdot)$,where

$$T_2(s) = \int_0^s u^{-3/2}\theta^{1/2}\big(u\log(1+1/u)/\log 2\big) f^{1/2}(u)du +$$
$$\int_0^s u^{-1-1/(2q)}[\log^{1/2-1/(2q)}(1+1/u)g^{1/q}(u)du.$$

For $x \in D[0,1]$ define $\Delta_x(s,t,u) = |x(s) - x(t)| \wedge |x(t) - x(u)|$. Let $Z = \{Z(t),\ \ t \in [0,1]\}$ be a separable cadlag random process defined on the probability space $(\Omega, \mathcal{F}, P)$. Let $h_1, h_2, \theta_1, \theta_2$ be increasing functions, $h_1(0) = h_2(0) = \theta_1(0) = \theta_2(0) = 0$ and θ_1, θ_2 are concave. The following result is proved in Bézandry and Fernique (1992).

Theorem 6 (see lemma 1.1.3 and theorem 1.2 in Bézandry and Fernique (1992)). Assume the process $Z = \{Z(t), t \in [0,1]\}$ is continuous from the right in L_1 at each $t \in [0,1]$. Assume for each $0 \le s \le t \le u \le 1$ and each $A \in \mathcal{F}$

$$\mathbf{E}|Z(s) - Z(t)| \wedge |Z(t) - Z(u)|\mathbb{I}_A \le h_1(u-s)\theta_1(\mathbf{P}(A)) + h_2(u-s)\theta_2(\mathbf{P}(A)).$$

Then there exists a sequence of random functions $\{f_k,\ k \ge 1\},\ \ f_k : \Omega \times [0,1] \to \mathbf{S}_k := \{j \cdot 2^{-k},\ \ 0 \le j \le 2^k\}$ satisfying:

1. For each $k \ge 1$ and each $\omega \in \Omega$ the function $f_k(\omega, \cdot) \to \mathbf{S}_k$ is non decreasing, $f_k(\omega, t) = t$ for $t \in \mathbf{S}_k$;

2. For each $k \ge 1$

$$\sup_{t\in[0,1]} |Z(\omega,t) - Z(\omega, f_k(\omega,t))| \le \sum_{m \ge k} \sum_{t \in \mathbf{S}_{m+1}\setminus \mathbf{S}_m} \Delta_{Z(\omega,\cdot)}(t - 2^{-k-1}, t, t + 2^{-k-1}) \quad (2.5)$$

and

$$\mathbf{E}\sup_{t\in[0,1]}|Z(\omega,t)-Z(\omega,f_k(\omega,t))|\le 2\int_0^{2^{-k}}u^{-2}(h_1(u)\theta_1(u)+h_2(u)\theta_2(u))du. \quad (2.6)$$

For a cadlag process Z satisfying conditions of Theorem 4 define

$$Z^{\{k\}}=\{Z(\omega,f_k(\omega,t)),\ t\in[0,1],\omega\in\Omega\},k\ge 1.$$

One may define the other approximate process

$$Z^{[k]}(\omega,t)=\sum_{i=0}^{2^k-1}Z(\omega,i\cdot 2^{-k})\mathbb{I}\{i\cdot 2^{-k}\le t<(i+1)\cdot 2^{-k}\}+Z(\omega,1)\mathbb{I}\{t=1\}.$$

Remark, that $\|Z^{\{k\}}\|=\max\{|Z(i\cdot 2^{-k}|,\ 0\le i\le 2^k\}=\|Z^{[k]}\|$ and hence

$\mathbf{\Delta}(\cdot,Z^{\{k\}},Y)=\mathbf{\Delta}(\cdot,Z^{[k]},Y)$.

Lemma 1. For each $k\ge 1$ and each $r,\varepsilon>0$

$$|\mathbf{P}(\|S_n\|<r)-\mathbf{P}(\|Y\|<r)|\le$$

$$I_1(S_n,\varepsilon)+3I_1(Y,\varepsilon)+2I_2(Y,\varepsilon)+2\cdot\mathbf{\Delta}(r,S_n^{\{k\}},Y^{[k]})+\mathbf{\Delta}(r-\varepsilon,S_n^{\{k\}},Y^{[k]}),$$

where $I_1(S_n,\varepsilon)=\mathbf{P}(\|S_n^{\{k\}}-S_n\|>\varepsilon), I_1(Y,\varepsilon)=\mathbf{P}(\|Y^{[k]}-Y\|>\varepsilon),\ I_2(Y,\varepsilon)=\mathbf{P}(\|Y^{[k]}\|\in[r-\varepsilon,r+\varepsilon])$.

Proof. The proof is standard, see e.g., Bentkus et al.(1990).

Proof of theorem 4. Let k be the integer satisfying $2^{k-1}<N_n\le 2^k$. Fix $r,\varepsilon>0$ and apply lemma 1 to the processes S_n and Y. It suffices to estimate quantities $I_1(S_n,\varepsilon),\ I_1(Y,\varepsilon),I_2(Y,\varepsilon),\mathbf{\Delta}(\cdot,S_n^{[k]},Y^{[k]})$, see also remark before the lemma 1. It follows from (2.6) and lemma 3 below that

$$I_1(S_n,\varepsilon)=\mathbf{P}(\|S_n^{\{k\}}-S_n\|\ge\varepsilon)\le\varepsilon^{-1}\mathbf{E}\|S_n^{\{k\}}-S_n\|\le 2c_2\varepsilon^{-1}T_1(2^{-k}). \quad (2.7)$$

Define the pseudometric $d(s,t)=\big(\mathbf{E}(Y(s)-Y(t))^2\big)^{1/2},\ \ s,t\in[0,1]$. Let $N(\delta),\ \delta>0$ be the minimal number of balls $B(u,\delta)=\{v\in[0,1]:\ d(u,v)<\delta\}\subset[0,1],\ u\in[0,1]$ that cover the interval $[0,1]$. By theorem 5.5 of Jain and Marcus (1978), there exists an absolute constant c such that for each $\varepsilon,\delta>0$

$$\mathbf{P}(\sup_{|u-v|<\delta}|Y(u)-Y(v)|>\varepsilon)\le c\varepsilon^{-1}(\int_0^\delta H^{1/2}(s)ds+\delta\log^{1/2}\log(1/\delta)), \quad (2.8)$$

where $H(\delta) = \log N(\delta)$. A simple calculation shows that condition (1.3) implies $g(u) \leq cu^{1/2}$. Hence,

$$\left(\mathbf{E}(Y(s) - Y(t))^2\right)^{1/2} = \left(\mathbf{E}(X(s) - X(t))^2\right)^{1/2} \leq$$

$$(\mathbf{E}|X(s) - X(t)|^q)^{1/q} \leq c|t - s|^{1/q},$$

and $N(\delta) \leq 1 + \delta^{-q}$. We have

$$I_1(Y, \varepsilon) = \mathbf{P}(\|Y - Y^{[k]}\| > \varepsilon) \leq c\varepsilon^{-1} 2^{-k} k^{1/2}. \tag{2.9}$$

Theorem 4 of Lifshits (1986) applies to the Gaussian process Y and yields

$$\sup_{\varepsilon > 0} \sup_{r \geq 0} \varepsilon^{-1} \mathbf{P}(\|Y\| \in [r, r + \varepsilon]) \leq C_5, \tag{2.10}$$

where $C_5 = C_5(Y)$. Hence, $I_2(Y, \varepsilon) \leq C_5 \varepsilon$. It remains to estimate the quantity $\mathbf{\Delta}(S_n^{[k]}, Y^{[k]})$. The problem is equivalent to the estimation of the rate of convergence in the CLT uniformly over the class of all balls with the center at the origin in the finitedimensional Banach space $l_\infty^{2^k}$, where $2^{k-1} < N_n \leq 2^k$. Using the standard technique due to Bentkus (1982) and Račkauskas (1984), see, e.g., Paulauskas and Račkauskas (1989), one may obtain the following bound

$$\mathbf{\Delta}(S_n^{[k]}, Y^{[k]}) \leq C(Y) n^{-1/6} \log^{2/3}(2^k)(1 + \mathbf{E}\|X\|^3), \tag{2.11}$$

if $2^k \geq N_n \geq n^{1/3}, \ n \geq 1$.

It follows from lemma 1 and the estimates of $I_1(S_n, \varepsilon)$, $I_1(Y, \varepsilon), I_2(Y, \varepsilon)$, $\mathbf{\Delta}(\cdot, S_n^{[k]}, Y^{[k]})$ that

$$\mathbf{\Delta}(r, S_n, Y) \leq C(Y) n^{-1/6} \log^{2/3}(N_n)(1 + \mathbf{E}\|X\|^3) +$$

$$C_5 \varepsilon + c_2 \varepsilon^{-1} T_1(N_n^{-1}) + c\varepsilon^{-1} N_n^{-1} \log^{1/2} \log(N_n).$$

Choosing $\varepsilon = \left(T_1(N_n^{-1}) + N_n^{-1} \log^{1/2} \log(N_n)\right)^{1/2}$ gives the desired bound (2.1). Theorem is proved.

Proof of Theorem 5. The proof is almost the same as that of theorem 4. Only now we use lemma 4 instead of lemma 3 when estimating

$$I_1(S_n,\varepsilon) \le \varepsilon^{-1}\mathbf{E}\|S_n^{\{k\}} - S_n\| \le 2c_3\varepsilon^{-1}T_2(2^{-k}), \tag{2.12}$$

by means of theorem 6. Theorem is proved.

Proof of theorem 2. If $p = 2,\ q \ge 2$ we apply theorem 4. It is easy to see that if $f(u) = cu^{1+\alpha}$ and $g(u) = cu^{1/2+\alpha}$ then

$$T_1(u) \le \int_0^u s^{-1+\alpha/2}ds + \int_0^u s^{-1+\alpha/q}\log^{1/2-1/(2q)}(1+1/s)ds \le c(\alpha,q)u^{\alpha/(2+q)},$$

$u \in [0,1]$. Choosing $N_n = n^\gamma$ with $\gamma \ge 2 + [2(2+q)/\alpha]$ gives $[T_1(N_n^{-1})]^{1/2} + [N_n^{-1}\log^{1/2}\log(N_n)]^{1/2} \le c(\alpha,q)n^{-1}$. Now (1.4) follows from (2.1).

If $p > 2, q \ge 2$ we apply theorem 5 with the functions $f(u) = cu^{2(1+\alpha)/p},\ \theta(u) = u^{1-2/p},\ q(u) = u^{1/2+\alpha}$. We have

$$T_2(u) \le \int_0^u s^{-1+\alpha/p}\log^{1/2-1/(2p)}(1+1/s)ds +$$

$$\int_0^u s^{-1+\alpha/q}\log^{1/2-1/(2q)}(1+1/s)ds \le c(\alpha,p,q)u^{\alpha/(p+q)}, u \in [0,1].$$

The bound (1.4) follows from (2.4) where $N_n = n^\gamma$ and $\gamma \ge 2 + [2(p+q)/\alpha]$. Theorem is proved.

Rest of the section is devoted to the non-uniform bound, i.e., the bound for $(1+r)^3\mathbf{\Delta}(r,S_n,Y)$.

Lemma 2 (see, e.g., lemma 4.1 in Bentkus et al. (1990)). For all $n,k \ge 1$ and $r > \varepsilon > 0$ the following inequality holds

$$\mathbf{\Delta}(r,S_n,Y)) \le$$

$$\mathbf{\Delta}(r-\varepsilon,S_n^{\{k\}},Y^{[k]}) + \mathbf{\Delta}(r,S_n^{\{k\}},Y^{[k]}) + I_3(S_n,\varepsilon,r) + I_3(Y,\varepsilon,r) + I_2(Y,\varepsilon),$$

where $I_3(S_n,\varepsilon,r) = \mathbf{P}(\|S_n^{\{k\}} - S_n\| > \varepsilon, \|S_n\| > r),\ I_3(Y,\varepsilon,r) = \mathbf{P}(\|Y^{[k]} - Y\| > \varepsilon, \|Y\| > r)$.

Proof of theorem 3. Denote $d = \min(p, \beta)$. Assume without loss of generality that $r > 10$. If $r \le 10$ then theorem follows from theorem 2. Fix integer k satisfying $2^k > n^{1/3}$ and $\varepsilon = n^{-1/6}$. By lemma 2, it suffices to estimate $I_2(Y, \varepsilon)$, $I_3(S_n, \varepsilon, r)$, $I_3(Y, \varepsilon, r)$ and $\Delta(r, S_n^{[k]}, Y^{[k]})$.

Using Holder inequality and combining (2.12) and lemma 5, we get

$$I_3(S_n, \varepsilon, r) = \mathbf{E}\mathrm{I\!I}\{\|S_n^{\{k\}} - S_n\| > \varepsilon\}\mathrm{I\!I}\{\|S_n\| > r\} \le$$

$$[\mathbf{P}(\|S_n^{\{k\}} - S_n\| > \varepsilon)]^{1-3/d}\mathbf{P}^{3/d}(\|S_n\| > r) \le c(\beta, p, q)(1+r)^{-3}[\varepsilon^{-1}T_1(2^{-k})]^{1-3/d}.$$

Analogously, by (2.9) and Chebyshev inequality,

$$I_3(Y, \varepsilon, r) \le [c\varepsilon^{-1}2^{-k}k^{1/2}]^{1/2}r^{-3}(\mathbf{E}\|Y\|^6)^{1/2}.$$

Here we use the fact that sample continuous Gaussian process Y has finite moments $\mathbf{E}\|Y\|^p < \infty$ for each $p > 0$.

Combining (2.10) with the estimates for the density function of the distribution of $\|Y\|$ due to Tsirel'son (1975), we get

$$I_2(Y, \varepsilon) \le C_6(1+r)^{-3}\varepsilon, \quad \forall r > \varepsilon > 0,$$

where $C_6 = C_6(Y)$ is an absolute constant. It remains to estimate $\mathbf{\Delta}(r, S_n^{[k]}, Y^{[k]})$. In what follows we apply theorem 4.5 from Bentkus et al. (1990). Define

$$M_3 = n^{-1/6}(1 \vee (\mathbf{E}\|X\|^3 + \mathbf{E}\|Y\|^3)^{1/3}); D_n(2^{-k}) = M_3 \vee W(2^{-k}, M_3).$$

$$W(2^{-k}, t) = \sup_{r \ge 0}(1+r)^3\mathbf{P}(\sup_{|u-v| \le 2^{-k}} |Y(u) - Y(v)| > t,\ \|Y\| > r);$$

By theorem 4.5 in Bentkus et al. (1990), there exists an absolute constant $C_7 = C_7(Y)$ such that for each $r > \varepsilon > 0$

$$\mathbf{\Delta}(r, S_n^{[k]}, Y^{[k]}) \le C_7(1+r)^{-3}\log^2(2^k)[D_n(2^{-k}) \vee D_n^3(2^{-k})].$$

Using Holder and Chebyshev inequalities and (2.8), we estimate

$$W(2^{-k}, t) \le \sup_{r \ge 0}(1+r)^{-3}\mathbf{P}^{1/2}(\sup_{|u-v| < 2^{-k}} |Y(u) - Y(v)| > t)\mathbf{P}^{1/2}(\|Y\| > r) \le$$

$$\sup_{r\ge 0}(1+r)^3[ct^{-1}2^{-k}k^{1/2}]^{1/2}r^{-3}\big(\mathbf{E}\|Y\|^6\big)^{1/2}\le C(Y)t^{-1/2}2^{-k/2}k^{1/4}.$$

A simple calculation gives

$$D_n(2^{-k})\vee D_n^3(2^{-k})\le n^{-1/6}(1+\mathbf{E}\|X\|^3+\mathbf{E}\|Y\|^3)+C(Y)n^{1/12}2^{-k/2}k^{1/4},$$

since $2^k\ge n^{1/3}$. Hence

$$\mathbf{\Delta}(r,S_n^{[k]},Y^{[k]})\le C(Y)(1+r)^{-3}k^2(n^{-1/6}(1+\mathbf{E}\|X\|^3+\mathbf{E}\|Y\|^3)+n^{1/12}2^{-k/2}k^{1/4}).$$

Combining the estimates of $I_2(Y,\varepsilon),\ I_3(S_n,\varepsilon,r),\ I_3(Y,\varepsilon,r)$ and $\Delta(r,S_n^{[k]},Y^{[k]})$ and lemma 2, we get

$$\Delta(r,S_n,Y)(1+r)^3\le$$
$$C(Y)\big(k^2[n^{-1/6}(1+\mathbf{E}\|X\|^3+\mathbf{E}\|Y\|^3)+n^{1/12}2^{-k/2}k^{1/4}]+$$
$$[\varepsilon^{-1}T_2(2^{-k})]^{1-3/d}+[\varepsilon^{-1}2^{-k}k^{1/2}]^{1/2}+\varepsilon\big). \tag{2.13}$$

Recall, that $T_2(2^{-k})\le c(\alpha,p,q)(2^{-k})^{\alpha/(p+q)}$, see proof of theorem 2. Choose $2^{-k}\le n^{-\gamma}$ where γ satisfies

$$\gamma>1/3;\ [\gamma(\alpha/(p+q))-1/6]\cdot(1-3/d)>1/6;,\gamma/2-1/12>1/6;\gamma-1/6>1/6.$$

Then the summ in the right hand side of (2.13) does not exceed

$$C(Y)k^2n^{-1/6}(1+\mathbf{E}\|X\|^3+\mathbf{E}\|Y\|^3).$$

Theorem is proved.

The argument used in proof of theorem 3 allows to get more general results. For instance the non uniform estimates (but with a slower speed of convergence with respect to n) can be obtained if instead of condition $f(u)\le cu^{1+\alpha},\ g(u)\le cu^{1/2+\alpha}$ one requires the finiteness of the integrals that appear in the proof of lemma 5.

Proof of the remark. It was mentioned in Bloznelis and Paulauskas (1993) that a transformation of the parameter set by means of increasing continuous function does not influence the quantity $\mathbf{\Delta}(\cdot,S_n,Y)$.

It is possible to construct a sample continuous process X which satisfies the CLT in $C[0,1]$ and for which the rate of order $n^{-1/6}$ can not be improved, cf. Bentkus et al (1990).

3. Auxiliary results

Lemma 3. Assume X satisfies conditions of theorem 4. Then there exists a constant $c_2 = c_2(q)$ such that for all $A \in \mathcal{F}_1$ and each $0 \le s \le t \le u \le 1$

$$\mathbf{E}|S_n(s) - S_n(t)| \wedge |S_n(t) - S_n(u)| \mathbb{I}_A \le$$

$$c_2 f^{1/2}(u-s)\mathbf{P}^{1/2}(A) + c_2 g^{1/q}(u-s)[\mathbf{P}(A)]^{1-1/(2q)} \log^{1/2-1/(2q)}(1+1/\mathbf{P}(A)).$$

Lemma 4. Assume X satisfies conditions of theorem 5. Then there exists a constant $c_3 = c_3(q)$ such for all $A \in \mathcal{F}_1$ and all $0 \le s \le t \le u \le 1$,

$$\mathbf{E}|S_n(s) - S_n(t)| \wedge |S_n(t) - S_n(u)| \mathbb{I}_A \le$$

$$c_3 f^{1/2}(u-s)\mathbf{P}^{1/2}(A)\theta^{1/2}(\mathbf{P}(A)\log(1+1/\mathbf{P}(A))/\log 2) +$$

$$c_3 g^{1/q}(u-s)\mathbf{P}^{1-1/(2q)}(A)\log^{1/2-1/(2q)}(1+1/\mathbf{P}(A)). \qquad (3.12)$$

The proof of lemmas 3 and 4 goes along the lines of the proof of lemmas 2.4.1, 2.4.2 and 2.4.4 of Bézandry and Fernique (1992), see also proof of theorem 2 in Bloznelis and Paulauskas (1993).

Lemma 5. Assume X satisfies conditions of theorem 3. Then there exist a constant $c_4 = c_4(X, p, q, \beta)$ such that for all $r \ge 1, n \ge 1$, $\mathbf{P}(\|S_n\| \ge 0) \le c_4 r^{-d}$, where $d = \min(\beta, p)$.

Proof. Put $\lambda_m = a/(m \log^2(m)), m \ge 1$ and let the constant a be such that $\sum_{m \ge 1} \lambda_m = 1$. By (2.5),

$$\|S_n^{\{1\}} - S_n\| \le \sum_{m \ge 1} \sum_{t \in \mathbf{S}_{m+1} \setminus \mathbf{S}_m} \Delta_{S_n}(t - 2^{-m-1}, t, t + 2^{-m-1}).$$

Hence, for each $r \ge 0$

$$\mathbf{P}(\|S_n\| \ge 2r) \le \mathbf{P}(\|S_n^{\{1\}}\| \ge r) +$$

$$\mathbf{P}(\sum_{m\geq 1}\sum_{t\in \mathbf{S}_{m+1}\setminus \mathbf{S}_m} \Delta_{S_n}(t-2^{-m-1},t,t+2^{-m-1})\geq r\sum_{m\geq 1}\lambda_m). \tag{3.2}$$

The standard argument yields

$$\mathbf{P}(\|S_n^{\{1\}}\|\geq r)=\mathbf{P}(\max(|S_n(0)|,|S_n(1/2)|,S_n(1)|)\geq r)\leq cr^{-\beta},$$

since $\mathbf{E}\|X\|^\beta<\infty$ and the process X is centered. The second summand in (3.2) does not exceed

$$\mathbf{I}:=\sum_{m\geq 1}\sum_{t\in \mathbf{S}_{m+1}\setminus \mathbf{S}_m}\mathbf{P}(\Delta_{S_n}(t-2^{-m-1},t,t+2^{-m-1})\geq r\lambda_m).$$

Below we will prove that for all $s\leq t\leq u$ and each $\lambda>0$

$$\mathbf{P}(\Delta_{S_n}(s,t,u)\geq\lambda)\leq$$

$$c(p,q)[\lambda^{-p}f(u-s)+\lambda^{-2q}g^2(u-s)+\lambda^{-2v}(f^{2v/p}(u-s)+g^{2v/q}(u-s))], \tag{3.3}$$

where $v=\max(p,q)$. Then

$$\mathbf{I}\leq\sum_{m\geq 1}2^m c(p,q)[r^{-p}(m\log^2(m))^p f(2^{-m})+r^{-2q}(m\log^2(m))^{2q}g^2(2^{-m})]+$$

$$\sum_{m\geq 1}2^m c(p,q)[r^{-2v}(m\log^2(m))^{2v}f^{2v/p}(2^{-m})+g^{2v/q}(2^{-m})]\leq$$

$$c'(p,q)\Big(r^{-p}\int_0^1 u^{-2}h^p(u)f(u)du+r^{-2q}\int_0^1 u^{-2}h^{2q}(u)g^2(u)du+$$

$$r^{-2v}\int_0^1 u^{-2}h^{2v}(u)\big(f^{2v/p}(u)+g^{2v/q}(u)\big)du\Big),$$

where $h(u)=\big(u\log(1+u^{-1})\big)$. These integrals are finite, since $f(u)\leq cu^{1+\alpha}$, $g(u)\leq cu^{1/2+\alpha}$. Hence $\mathbf{I}\leq c''(p,q)(r^{-p}+r^{-2q}+r^{-2v})$ and the lemma follows.

It remains to prove (3.3). Let $X_1,X_2,...,X_n,X_1',X_2',...,X_n'$ be independent copies of X. Let $\varepsilon,\varepsilon_1,\varepsilon_2,...$ be i.i.d. Bernoulli random variables $\big(\mathbf{P}(\varepsilon=+1)=\mathbf{P}(\varepsilon=-1)=1/2\big)$. Assume the sequences $\{X_n\}$, $\{X_n'\}$, $\{\varepsilon_n\}$ are independent. Define $X^s=X'-X$ and $S_n^s=S_n-S_n'$, where $S_n'=n^{-1/2}(X_1'+\cdots+X_n')$.

Let $\overline{a} = S_n(s) - S_n(t), \overline{b} = S_n(t) - S_n(u), \overline{a}' = S_n'(s) - S_n'(t), \overline{b}' = S_n'(t) - S_n'(u)$. A simple inequality

$$|\overline{a}| \wedge |\overline{b}| \leq |\overline{a} - \overline{a}'| \wedge |\overline{b} - \overline{b}'| + |\overline{a}'| \wedge |\overline{b}| + |\overline{a}| \wedge |\overline{b}'| \tag{3.4}$$

yields

$$\mathbf{P}(|\overline{a}| \wedge |b| > 3\lambda) \leq$$

$$\mathbf{P}(|\overline{a} - \overline{a}'| \wedge |\overline{b} - \overline{b}'| > \lambda) + \mathbf{P}(|\overline{a}'| \wedge |\overline{b}| > \lambda) + \mathbf{P}(|\overline{a}| \wedge |\overline{b}'| > \lambda) = \mathbf{P}_1 + \mathbf{P}_2 + \mathbf{P}_3.$$

The second and the third probabilities are estimated in the same way. Let us estimate the second one,

$$P_2 \leq \lambda^{-2q} \mathbf{E}|S_n(s) - S_n(t)|^q |S_n'(t) - S_n'(u)|^q =$$

$$\lambda^{-2q} \mathbf{E}|S_n(s) - S_n(t)|^q \mathbf{E}|S_n'(t) - S_n'(u)|^q \leq$$

$$c(q)\lambda^{-2q} \mathbf{E}|X(s) - X(t)|^q \mathbf{E}|X(t) - X(u)|^q \leq c(q)\lambda^{-2q} g^2(u - s).$$

Analogously $P_3 \leq c(q)\lambda^{-2q} g^2(u - s)$. Let us estimate P_1. Denote

$$x = n^{-1/2}(X^s(s) - X^s(t)), y = n^{-1/2}(X^s(t) - X^s(u)), m = |x| \wedge |y|,$$

$$\alpha = sign(x), \beta = sign(y), u = |x| - m, v = |y| - m.$$

Then

$$\mathbf{P}_1 = \mathbf{P}(|\sum_{i=1}^n \varepsilon_i \alpha_i x_i| \wedge |\sum_{i=1}^n \varepsilon_i \beta_i y_i| \geq \lambda) \leq \mathbf{P}(|\sum_{i=1}^n \varepsilon_i \alpha_i m_i| \geq \lambda/3) +$$

$$\mathbf{P}(|\sum_{i=1}^n \varepsilon_i \beta_i m_i| \geq \lambda/3) + \mathbf{P}(|\sum_{i=1}^n \varepsilon_i \alpha_i u_i| \wedge |\sum_{i=1}^n \varepsilon_i \beta_i v_i| \geq \lambda/3) \leq \mathbf{P}_4 + \mathbf{P}_5 + \mathbf{P}_6.$$

The last probability $\mathbf{P}_6$ does dot exceed

$$(\lambda/3)^{-2q} \mathbf{E}(|\sum_{i=1}^n \varepsilon_i \alpha_i u_i| \wedge |\sum_{i=1}^n \varepsilon_i \beta_i v_i|)^{2q} \leq$$

$$c(q)(\lambda/3)^{-2q} \mathbf{E}|X^s(s) - X^s(t)|^q \mathbf{E}|X^s(t) - X^s(u)|^q,$$

see proof of theorem 2 in Bloznelis and Paulauskas (1993 b). It follows from (1.2) that $\mathbf{P}_6 \leq c(q)(\lambda)^{-2q} g^2(u-s)$. It remains to estimate probabilities $\mathbf{P}_4$ and $\mathbf{P}_5$. In what follows we apply the iterated Hoffmann-Jorgensen (1975) inequality. For each $v \geq 1$ there exist constants $c_1(v),\ c_2(v),\ c_3(v)$ such that for each $\lambda > 0$ and $n \geq 1$

$$\mathbf{P}(|\sum_{i=1}^n \varepsilon_i \alpha_i m_i| \geq c_1(v)\lambda) \leq c_2(v) \sum_{i=1}^n \mathbf{P}(|m_i| \geq \lambda) + c_3(v)\mathbf{P}^v(|\sum_{i=1}^n \varepsilon_i \alpha_i m_i| \geq \lambda). \tag{3.5}$$

Denote $a = n^{-1/2}(X(s) - X(t)),\ b = n^{-1/2}(X(t) - X(u)),\ a' = n^{-1/2}(X'(s) - X'(t)),\ b' = n^{-1/2}(X'(t) - X'(u))$. Inequality (3.4) with $a,\ b,\ a',\ b'$ instead of $\overline{a},\ \overline{b},\ \overline{a}',\ \overline{b}'$ yields

$$\mathbf{P}(|m| > 4\lambda) = \mathbf{P}(|a - a'| \wedge |b - b'| > 4\lambda) \leq$$

$$\lambda^{-p}(\mathbf{E}(|a| \wedge |b|) + \mathbf{E}(|a'| \wedge |b'|)) + \lambda^{-2q}(\mathbf{E}(|a'| \wedge |b|)^{2q} + \mathbf{E}(|a| \wedge |b'|)^{2q}) \leq$$

$$n^{-p/2}\lambda^{-p} 2f(u-s) + n^{-q}\lambda^{-2q} 2g^2(u-s). \tag{3.6}$$

On the other hand

$$\mathbf{P}(|\sum_{i=1}^n \varepsilon_i \alpha_i m_i| \geq \lambda) \leq \lambda^{-2} \sum_{i=1}^n \mathbf{E}|m_i|^2 \leq \lambda^{-2} c(f^{2/p}(u-s) + g^{2/q}(u-s)). \tag{3.7}$$

In the last inequality we estimate m by means of (3.4) but with $a,\ b,\ a',\ b'$ instead of $\overline{a},\ \overline{b},\ \overline{a}',\ \overline{b}'$ and apply Holder inequality. Combining (3.6), (3.7) and (3.5) with $v = \max(p, q)$ we get

$$\mathbf{P}_4 \leq c\big(\lambda^{-p} f(u-s) + \lambda^{-2q} g^2(u-s) + \lambda^{-2v}(f^{2v/p}(u-s) + g^{2v/q}(u-s))\big).$$

Analogous estimation holds also for $\mathbf{P}_5$. Combining estimates of $\mathbf{P}_1 \leq \mathbf{P}_4 + \mathbf{P}_5 + \mathbf{P}_6$ and $\mathbf{P}_2,\ \mathbf{P}_3$ gives (3.3). Lemma is proved

4. Appendix

Here we prove lemmas 3 and 4.

Lemma 6. Assume X satisfies conditions of theorem 4. Then there exists a constant c depending only on q such that for each $A \in \mathcal{F}$ and all $0 \leq s \leq t \leq u \leq 1$

$$\mathbf{E}|S_n^s(s) - S_n^s(t)| \wedge |S_n^s(t) - S_n^s(u)| \mathbb{I}_A \leq$$

$$cf^{1/2}(u-s)+cg^{1/q}(u-s)[\mathbf{P}(A)]^{1-1/(2q)}\log^{1/2-1/(2q)}(1+1/\mathbf{P}(a)). \qquad (4.1)$$

Proof. Denote

$$x=n^{-1/2}(X^s(s)-X^s(t)), y=n^{-1/2}(X^s(t)-X^s(u)), m=|x|\wedge|y|,$$

$$\alpha=sign(x), \beta=sign(y), u=|x|-m, v=|y|-m.$$

It suffices to estimate

$$\mathbf{E}\{|\sum_{i=1}^n \varepsilon_i\alpha_i x_i|\wedge|\sum_{i=1}^n \varepsilon_i\beta_i y_i|\mathbb{I}_A\}\le$$

$$\mathbf{E}|\sum_{i=1}^n \varepsilon_i\alpha_i m_i|\mathbb{I}_A+\mathbf{E}|\sum_{i=1}^n \varepsilon_i\beta_i m_i|\mathbb{I}_A+\mathbf{E}\{|\sum_{i=1}^n \varepsilon_i\alpha_i u_i|\wedge|\sum_{i=1}^n \varepsilon_i\beta_i v_i|\mathbb{I}_A\}. \qquad (4.2)$$

The last expectation is estimated in Bloznelis and Paulauskas (1993 b). We have

$$\mathbf{E}\{|\sum_{i=1}^n \varepsilon_i\alpha_i u_i|\wedge|\sum_{i=1}^n \varepsilon_i\beta_i v_i|\mathbb{I}_A\}\le cg^{1/q}(u-s)[\mathbf{P}(A)]^{1-1/(2q)}. \qquad (4.3)$$

The first and the second expectations are estimated in the same way. Let us estimate the first one,

$$\mathbf{E}|\sum_{i=1}^n \varepsilon_i\alpha_i m_i|\mathbb{I}_A\le\big(\mathbf{E}|\sum_{i=1}^n \varepsilon_i\alpha_i m_i|^2\mathbb{I}_A\big)^{1/2}[\mathbf{P}(A)]^{1/2}=$$

$$\big(\mathbf{E}(\mathbf{E}_2|\sum_{i=1}^n \varepsilon_i\alpha_i m_i|^2\mathbb{I}_A)\big)^{1/2}[\mathbf{P}(A)]^{1/2}. \qquad (4.4)$$

Here $\mathbf{E}_2$ denotes the expectation taken with respect to $\mathbf{P}_2$. Using the standard Orlicz space technique, see Bézandry and Fernique (1992), one obtains

$$E_2|\sum_{i=1}^n \varepsilon_i\alpha_i m_i|^2\mathbb{I}_A\le c\sum_{i=1}^n m_i^2\mathbf{P}_2(A)\log(1+1/\mathbf{P}_2(A)). \qquad (4.5)$$

Denote $\xi(u)=u\log(1+1/u)/\log 2$. Put

$$a=n^{-1/2}(X(s)-X(t)),\quad a'=n^{-1/2}(X'(s)-X'(t)),\quad b=n^{-1/2}(X(t)-X(u))$$

$$b'=n^{-1/2}(X'(t)-X'(u)),\quad d=|a|\wedge|b|,\quad d'=|a'|\wedge|b'|.$$

A simple inequality

$$|m| = |a - a'| \wedge |b - b'| \leq d + d' + |a'| \wedge |b| + |a| \wedge |b'|$$

yields

$$\mathbf{E}|m|^2\xi(\mathbf{P}_2(A)) \leq 4\mathbf{E}d^2\xi(\mathbf{P}_2(A)) + 4\mathbf{E}d'^2\xi(\mathbf{P}_2(A)) +$$

$$4\mathbf{E}|a'|^2 \wedge |b|^2\xi(\mathbf{P}_2(A)) + 4\mathbf{E}|a|^2 \wedge |b'|^2\xi(\mathbf{P}_2(A)). \tag{4.6}$$

We have

$$\mathbf{E}d^2\xi(\mathbf{P}_2(A)) \leq \mathbf{E}d^2 \leq n^{-1}f(u-s), \mathbf{E}d'^2\xi(\mathbf{P}_2(A)) \leq \mathbf{E}d'^2 \leq n^{-1}f(u-s). \tag{4.7}$$

By lemma 1.1.3 of Fernique (1981),

$$\mathbf{E}|a'|^2 \wedge |b|^2\xi(\mathbf{P}_2(A)) \leq n^{-1}g^{2/q}(u-s)[\mathbf{E}\xi(\mathbf{P}_2(A))]^{1-1/q}$$

provided

$$\mathbf{E}|a'|^2 \wedge |b|^2 \mathbb{I}\{|a'|^2 \wedge |b|^2 \geq M\} \leq n^{-1}g^{2/q}(u-s)[\mathbf{P}(|a'|^2 \wedge |b|^2 \geq M)]^{1-1/q}, \forall M \geq 0.$$

Observe, that the last inequality follows from the independence of $|a'|$ and $|b|$, Holder inequality and condition (2.1) of theorem 4. We have

$$\mathbf{E}|a'|^2 \wedge |b|^2\xi(\mathbf{P}_2(A)) \leq n^{-1}g^{2/q}(u-s)[\mathbf{E}\xi(\mathbf{P}_2(A))]^{1-1/q}$$

$$\leq n^{-1}g^{2/q}(u-s)[\xi(\mathbf{P}(A))]^{1-1/q}, \tag{4.9}$$

since, by Iensen inequality, $\mathbf{E}\xi(\mathbf{P}_2(A)) \leq \xi(\mathbf{P}(A))$. Analogously,

$$\mathbf{E}|a|^2 \wedge |b'|^2\xi(\mathbf{P}_2(A)) \leq n^{-1}g^{2/q}(u-s)[\xi(\mathbf{P}(A))]^{1-1/q}. \tag{4.10}$$

Combining inequalities (4.2,4,5,6) and estimates (4.3,7,9,10) we get (4.1). Lemma is proved.

Lemma 3. Assume X satisfies conditions of theorem 4. Then there exist a constant c depending only on q such that for all $A \in \mathcal{F}_1$ and each $0 \leq s \leq t \leq u \leq 1$

$$\mathbf{E}|S_n(s) - S_n(t)| \wedge |S_n(t) - S_n(u)|\mathbb{I}_A \leq$$

$$cf^{1/2}(u-s)+cg^{1/q}(u-s)[\mathbf{P}(A)]^{1-1/(2q)}\log^{1/2-1/(2q)}(1+1/\mathbf{P}(A)). \qquad (4.11)$$

Proof. Let $x=S_n(s)-S_n(t), y=S_n(t)-S_n(u)$, $x'=S'_n(s)-S'_n(t), y'=S'_n(t)-S'_n(u)$. A simple inequality

$$|x|\wedge|y|\le|x-x'|\wedge|y-y'|+|x'|\wedge|y|+|x|\wedge|y'|$$

yields

$$\mathbf{E}|x|\wedge|y|\mathbb{I}_A\le\mathbf{E}|x-x'|\wedge|y-y'|\mathbb{I}_A+\mathbf{E}|x'|\wedge|y|\mathbb{I}_A+\mathbf{E}|x|\wedge|y'|\mathbb{I}_A. \qquad (4.12)$$

The first summand is estimated in lemma 6. Let us estimate the last two expectations. We have

$$\mathbf{E}|x'|\wedge|y|\mathbb{I}_A\le\mathbf{E}|x'|\mathbb{I}_A=\mathbf{E}|x'|\mathbf{E}\mathbb{I}_A=\mathbf{E}|S'_n(s)-S'_n(t)|\mathbf{P}(A),$$

since $\mathbb{I}_A$ and S'_n are independent. Further,

$$\mathbf{E}|S'_n(s)-S'_n(t)|\le\big(\mathbf{E}|S'_n(s)-S'_n(t)|^2\big)^{1/2}=\big(\mathbf{E}(X(s)-X(t))^2\big)^{1/2}$$

$$\le\big(\mathbf{E}|X(s)-X(t)|^q\big)1/q\le g^{1/q}(u-s).$$

Hence $\mathbf{E}|x'|\wedge|y|\mathbb{I}_A\le g^{1/q}(u-s)\mathbf{P}(A)$. Analogously, $\mathbf{E}|x|\wedge|y'|\mathbb{I}_A\le g^{1/q}(u-s)\mathbf{P}(A)$. Substitution of these estimates and (4.1) in (4.12) gives (4.11).Lemma is proved.

Lemma 4. Assume X satisfies conditions of theorem 5. Then there exists a constant C depending on q such for all $A\in\mathcal{F}_1$ and all $0\le s\le t\le u\le 1$,

$$\mathbf{E}|S_n(s)-S_n(t)|\wedge|S_n(t)-S_n(u)|\mathbb{I}_A\le$$

$$Cf^{1/2}(u-s)\mathbf{P}^{1/2}(A)\theta^{1/2}(\mathbf{P}(A)\log(1+1/\mathbf{P}(A))/\log 2)$$

$$+Cg^{1/q}(u-s)\mathbf{P}^{1-1/(2q)}(A)\log^{1/2-1/(2q)}(1+1/\mathbf{P}(A)). \qquad (4.13)$$

Proof. The proof goes along the lines of the argument we used in proof of Lemmas 6 and 3. The only difference is that now expectations $\mathbf{E}d^2\xi(\mathbf{P}_2(A))$ and $\mathbf{E}d'^2\xi(\mathbf{P}_2(A))$ are estimated as in proof of lemma 2.4.1 in Bézandry and Fernique (1992),

$$\mathbf{E}d^2\xi(\mathbf{P}_2(A))\le cf(u-s)\theta(\xi[\mathbf{P}(A)\log(1+1/\mathbf{P}(A))/\log 2]),$$

$$\mathbf{E}d'^2\xi(\mathbf{P}_2(A)) \leq cf(u-s)\theta(\xi[\mathbf{P}(A)\log(1+1/\mathbf{P}(A))/\log 2]).$$

The rest of the proof of the lemma coincides with that of lemmas 6 and 3. Lemma is proved.

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