CENTRAL LIMIT THEOREM
FOR STOCHASTICALLY CONTINUOUS PROCESSES.
CONVERGENCE TO STABLE LIMIT (REVISED).

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ABSTRACT. Let $X = \{X(t), \quad t \in [0,1]\}$ be a stochastically continuous cadlag process. Assume that the k dimensional finite joint distributions of X are in the domain of normal attraction of a strictly p-stable, $0 < p < 2$, measure on R^k for all $1 \le k < \infty$. For functions f, g such that $\Lambda_p\big(|X(s) - X(u)|\big) < g(u-s)$ and $\Lambda_p\big(|X(s) - X(t)| \wedge |X(t) - X(u)|\big) < f(u-s)$, $0 \le s \le t \le u \le 1$, conditions are found which imply that the distributions $\mathcal{L}\big(n^{-1/p}(X_1 + \cdots + X_n)\big)$, $n \ge 1$, converge weakly in $D[0,1]$ to the distribution of a p-stable process. Here $X_1, X_2, ...$ are independent copies of X and $\Lambda_p(Z) = \sup_{t>0} t^p \mathbf{P}\{|Z| > t\}$ denotes the weak p-th moment of a random variable Z.

Key words. *Central limit theorem, Cadlag processes, Stable laws*

1. INTRODUCTION AND STATEMENT OF RESULTS.

Let $X = \{X(t), \quad t \in [0,1]\}$ be a stochastically continuous random process with sample paths in $D[0,1]$ (the space of real cadlag functions, i.e., functions which are right continuous and have left limits). We say that X satisfies the central limit theorem with index p in the space $D[0,1]$ (in short $X \in CLT_p(D)$), $0 < p < 2$, if the distributions $\mathcal{L}(S_n)$ of $S_n = n^{-1/p}(X_1 + \cdots + X_n)$, $n \ge 1$, converge weakly in $D[0,1]$ endowed with the Skorohod topology to the distribution of a p-stable cadlag process. Here $X_1, X_2, ...$ are independent copies of X.

In the case when p=2, i.e., when the limiting process is Gaussian, the central limit theorem in $D[0,1]$ was considered by a number of authors: Hahn(17), Giné and Zinn(12), Juknevičiene(19), Paulauskas and Stieve(23), Bézandry and Fernique(5), Bloznelis and Paulauskas(7,8), Fernique(11), etc. For $p < 2$, the CLT$_p$(D) is less investigated. Giné and Marcus(13) considered CLT$_p$(D) of stochastic integrals with respect to Levy processes. Dehling et al(9) proved CLT$_p$(D) for empirical processes weighted by random variables that belong to the domain of normal attraction of a p-stable law, see Example 2 below. Most of the papers devoted to the CLT with a p-stable limit, $0 < p \le 2$, deal with the processes whose sample paths are continuous or belong to some Banach space: Marcus and Woyczyński(21), Araujo and Giné(2), Marcus and Pisier(22), etc. In the present paper we prove the CLT for stochastically continuous processes in the case when the limiting process is p-stable, $0 < p < 2$, and cadlag.

Usually, cf. Ref. 5, 8, 11, conditions for X to satisfy CLT$_2$(D) are formulated in terms of moments of increments $|X(s) - X(u)|$ and $\big|X(s) - X(t)\big| \wedge \big|X(t) - X(u)\big|$, $s \le t \le u$. It is well known, see, e.g. Feller(10), that if a random variable Z is in the domain of normal attraction of a p-stable law then the weak p-th moment

$$\Lambda_p(Z) = \sup_{t>0} t^p \mathbf{P}(|Z| > t)$$

is finite, whereas $\mathbf{E}|Z|^p = +\infty$. Therefore, we shall use weak moments of increments.

Our aim is to obtain sufficient conditions for X to satisfy $\mathrm{CLT}_p(\mathrm{D})$ in terms of $\Lambda_p(X(u)-X(s))$ and $\Lambda_p(|X(s)-X(t)| \wedge |X(t)-X(u)|)$.

Let f, g be nonnegative increasing functions such that $f(0) = g(0) = 0$. We shall assume that

$$\Lambda_p(X(s)-X(u)) \le g(u-s), \quad 0 \le s \le u \le 1, \tag{1.1}$$

and

$$\Lambda_p(|X(s)-X(t)| \wedge |X(t)-X(u)|) \le f(u-s), \quad 0 \le s \le t \le u \le 1. \tag{1.2}$$

Theorem 1. Let $1 < p < 2$. Suppose that $X = \{X(t),\ t \in [0,1]\}$ is random process such that

(i) the k dimensional finite joint distributions of X are in the domain of normal attraction of a strictly p-stable measure on R^k for all $1 \le k < \infty$;

(ii) conditions (1.1) and (1.2) hold with f, g such that

$$\int_0 u^{-1-1/p} f^{1/p}(u) du < \infty \tag{1.3}$$

and

$$\int_0 u^{-1-1/(2p)} g^{1/p}(u) du < \infty. \tag{1.4}$$

Then the process X has a version X' with sample paths in $D[0,1]$ and $X' \in CLT_p(D)$.

Here and in what follows $c, c_1, c_2, \dots$ denote generic absolute constants. We shall write $c(T_1, T_2, \dots)$ when the constant depends on $T_1, T_2, \dots$.

Notice that in Theorem 1 we do not assume that X is stochastically continuous and cadlag. These two properties are ensured by the conditions (1.1-2) and (1.3-4). Condition (1.1) - (1.4) implies stochastic continuity (st.c.). Furthermore, condition (1.2) - (1.3) yields that st.c. process X has a cadlag version, say X'.

In Example 1 we show that condition (1.3) is close to the optimal for $1 < p < 2$. Unfortunately, we are not able to check optimality of the condition (1.4).

Next we give sufficient conditions for $CLT_p(D)$ in the case when $0 < p < 1$.

Theorem 2. Let $0 < p < 1$. Assume that X satisfies condition (i) of Theorem 1. If conditions (1.1) and (1.2) hold with f, g such that

$$f(t) \le c|t| \ln^{-(p+1+\varepsilon)}(1+|t|^{-1}) \tag{1.5}$$

and

$$g(t) \le c|t|^{1/2} \ln^{-(p+1/2+\varepsilon)}(1+|t|^{-1}). \tag{1.6}$$

then X has a version X' with sample paths in $D[0,1]$ and $X' \in CLT_p(D)$.

Bass and Pyke[3] proved a central limit theorem for triangular arrays of independent random variables with values in the space of set-indexed functions that are outer continuous with inner limits (a generalization of $D[0,1]$). An application of this general result yields sufficient conditions for $X \in CLT_p(D)$, but these conditions are uncomparable with the conditions of Theorems 1 and 2.

Proposition 1. Assume that X is symmetric, i.e. $\mathcal{L}(X) = \mathcal{L}(-X)$. Suppose that X satisfies conditions of Theorem 2, but with $p = 1$. Then X has a version X' with sample paths in $D[0,1]$ and $X' \in CLT_1(D)$.

Notice that conditions (1.3-4) are sharper than (1.5-6). This is because for $1 < p < 2$, we apply precise weak compactness criteria (of probability measures on $D[0,1]$) due to Bezandry and Fernique[5]. This criteria is not applicable to the case $0 < p \le 1$.

Example 1 (cf. Hahn[15,16]). Let $1 < p < 2$. Put $\big(\Omega, \mathbf{P}\big) = \big([0,1], \lambda\big)$, where λ denotes the Lebesgue measure. For $k \ge 1$, on the probability space $(\Omega, \mathbf{P})$ define the process

$$\eta_k(t,\omega) = \sum_{i=1}^{\infty} \min\big\{n^{2/p}, \, \ln_k\big(c(k)|t-\omega|\big)\big\} \mathbb{I}\big\{(n+1)^{-1} < \omega < n^{-1}\big\},$$

$\omega \in \Omega$, $t \in [0,1]$. Here the function $\ln_k$ is defined by induction:

$$\ln_0(x) = |x| \qquad \text{and} \qquad \ln_k(x) = \big|\ln_{k-1}|x|\big|, \quad k \ge 1.$$

We choose the constant $c(k)$ small enough to satisfy

$$\ln_r\big(c(k)|t-\omega|\big) > 1000, \quad t \in [0,1], \quad \omega \in \Omega, \quad t \ne \omega, \quad 1 \le r \le k.$$

Consider the process

$$X = \theta_p \, \eta_k,$$

where θ_p is a standard p-stable random variable $\Big(\mathbf{E}\exp\{it\theta_p\} = \exp\{-|t|^p\}\Big)$ independent of η_k. It is easy to see that the process X is symmetric, sample continuous and satisfies condition (i) of Theorem 1. Moreover, calculations show that conditions (1.1-2) are satisfied with the functions

$$f(u) = c(k,p)\, h_{k,p,0}(u) \qquad \text{and} \qquad g(u) = c(k,p)\, h_{k,p,0}(u),$$

where

$$h_{k,p,\varepsilon}(u) = |u| \Big(\ln_{k-1}^{1+\varepsilon}(u) \, \ln_{k-2}(u) \cdots \ln_1(u)\Big)^{-p}, \; k \ge 2, \;\; \text{and} \;\; h_{1,p,\varepsilon}(u) = |u|, \; \varepsilon \ge 0.$$

Obviously, g satisfies (1.4) and f fails to satisfy (1.3). Furthermore, we have $\mathbf{E}\|\eta_k\|^p = \infty$. Here $\|x\| = \sup_{t\in[0,1]} |x(t)|$ denotes the norm of an element x of the Banach space $C[0,1]$. Hence, by Lemma 2.1 of Ref. 14, $\mathcal{L}\big(n^{-1/p}(X_1 + \cdots + X_n)\big)$ does not converge weakly in the space $C[0,1]$ and therefore $X \notin CLT_p(D)$. Another proof of the fact that $X \notin CLT_p(D)$ could be obtained using the same argument as in Hahn[16].

Observe that condition (1.3) is close to the optimal since $f(u) = h_{k,p,\varepsilon}(u)$ satisfies (1.3), for each $\varepsilon > 0$.

In the next example an application of Theorem 1 yields the weak convergence (in $D[0,1]$) of weighted empirical processes.

Example 2. Let $U, U_1, U_2, \ldots$ be i.i.d. r. variables uniformly distributed in $[0,1]$. Let $V, V_1, V_2, \ldots$ be i.i.d. r. variables independent of the sequence $\{U_i,\ i \geq 1\}$ such that V belongs to the domain of normal attraction of some strictly p-stable law. Define an empirical process with random weights

$$F_n(t) = n^{-1/p} \sum_{i=1}^{n} w(t)\, V_i\, \mathrm{I\!I}\{U_i \leq t\}, \quad n \geq 1.$$

Here w is a nonrandom function. In Ref. 9 the weak convergence of $\mathcal{L}(F_n)$ was obtained in the case when $w \equiv 1$ and $1 < p < 2$. If $w \equiv 1$ then an application of Theorems 1 and 2 yields the weak convergence of $\mathcal{L}(F_n)$ for $0 < p < 2$, $p \neq 1$. Proposition 1 applies to the case $p = 1$, $\omega \equiv 1$ provided that V is symmetric, i.e., $\mathcal{L}(V) = \mathcal{L}(-V)$. Moreover, if $1 < p < 2$ and $w(t) = t^{-1/p}m(t)$, $t \in (0,1]$, where m is positive, continuous and nondecreasing function, then Theorem 1 yields the weak convergence of $\mathcal{L}(F_n)$ provided that, for some $\delta > 1/p$, $m(t) = O\big(\ln^{-\delta}(t^{-1})\big)$ as $t \to 0$, $t > 0$.

Remark 1. Let $Y = \{Y(t),\ t \in [0,1]\,\}$ be a Levy process with the parameter $p \in (0,\, 2)$ (p-stable process with independent increments). It is easy to see that for each $q < 2p$,

$$\mathbf{E}\big(|Y(t) - Y(s)| \wedge |Y(u) - Y(t)|\big)^q < \infty$$

Hence, it could be reasonable to consider the condition

$$\mathbf{E}\big(|X(t) - X(s)| \wedge |X(u) - X(t)|\big)^q < f(u - s)$$

with some $q < 2p$ instead of (1.2).

Remark 2. Theorems 1, 2 and Proposition 1 remain true if $f(u-s)$ and $g(u-s)$ in the right-hand sides of (1.1-2) are replaced by $f\big(F(u) - F(s)\big)$ and $g\big(G(u) - G(s)\big)$, where F, G are increasing continuous functions.

2. PROOFS.

In the proofs we use the technique developed in Ref. 5. The step towards the non Gaussian limit case is made using several facts from the theory of stable type Banach spaces.

A Banach space $(E, \|\cdot\|)$ is said to be of stable type p if for every sequence $(x_n) \subset E$ the convergence of $\sum \|x_n\|^p$ implies the convergence of $\sum g_n x_n$ almost surely, where the g_n's are independent standard p-stable random variables.

Theorem 3 (Theorem 1 in Ref. 25, see also Ref.21). A Banach space E is of stable type p for $0 < p < 2$, if and only if there exists a constant $C > 0$ such that

$$\Lambda_p(\|\sum_{i=1}^{n} Z_i\|) \leq C \sum_{i=1}^{n} \Lambda_p(\|Z_i\|) \tag{2.1}$$

for all symmetric independent E-valued r. variables $Z_1, Z_2, ..., Z_n$ such that $\Lambda_p(\|Z_i\|) < \infty,\ i = 1, 2, ..., n,\ n \geq 1$.

Remark 3. If $p > 1$ then (2.1) holds for non symmetric but centered r. variables as well, see the Remark after Theorem 4.12 in Ref. 24. An inspection of the proof of Theorem 1 in Ref. 25 shows that (2.1) holds for non-symmetric random variables if $p<1$.

In what follows Theorem 3 is applied to $E=R$. It seems that similar results to Theorems 1 and 2 could be formulated for E-valued cadlag processes (processes with sample paths in $D([0,1],E)$), where E is a Banach space of stable type, e.g. Hilbert space.

In what follows we present some known results concerning weak moments and two criteria of the weak compactness of sequences of probability measures on $D[0,1]$. Denote

$$N_p(\|Z\|)=\sup\big\{\mathbf{P}(A)^{-1/q}\int_A\|Z\|d\mathbf{P}\ ;\ A\in\mathcal{F},\ \mathbf{P}(A)>0\big\},\quad p>1.$$

Then for $q=p/(p-1)$

$$\big(\Lambda_p(\|Z\|)\big)^{1/p}\le N_p(\|Z\|)\le q\big(\Lambda_p(\|Z\|)\big)^{1/p}, \tag{2.2}$$

see e.g. Chapter 5. in Ref. 20.

Lemma 1 (Hoffmann-Jorgensen, see Ref. 18). Let $X_1,...,X_n$ be independent symmetric a Banach space valued random variables. Then for all $s,t>0$,

$$\mathbf{P}(\|X_1+\cdots+X_n\|>2t+s)\le\mathbf{P}(\sup_{1\le i\le n}\|X_i\|>s)+\mathbf{P}^2(\|X_1+\cdots+X_n\|>t), \tag{2.3}$$

where $\|\cdot\|$ denotes the norm of the Banach space.

Denote

$$\Delta_x(s,t,u)=|x(s)-x(t)|\wedge|x(t)-x(u)|,\quad x\in D[0,1],\quad 0\le s\le t\le u\le 1.$$

The following theorem is a simple corollary of Theorem 1.3 in Ref. 5.

Theorem 4. Let $p,q>1$. Let $\{X_n,\ n\ge 1\}$ be a sequence of stochastically continuous processes defined on a probability space $(\Omega,\mathcal{F},P)$ with sample paths in $D[0,1]$. Assume that the sequence $\{X_n,\ n\ge 1\}$ is equicontinuous in probability at the points $\{0;1\}$ and, for each $t\in[0,1]$, the sequence of real random variables $\{X_n(t),\ n\ge 1\}$ is stochastically bounded. Assume that there exist nondecreasing functions f, g with $f(0)=g(0)=0$ such that

$$\int_0(u^{-1-1/p}f^{1/p}(u)+u^{-1-1/q}g^{1/q}(u))du<\infty.$$

If, for each $n\ge 1$,

$$\forall\ 0\le s\le t\le u\le 1\qquad\text{and}\qquad\forall\ A\in\mathcal{F},$$

$$\mathbf{E}\,\Delta_{X_n}(s,t,u)\,\mathbb{I}\{A\}\le f^{1/p}(u-s)P(A)^{1-1/p}+g^{1/q}(u-s)P(A)^{1-1/q}$$

then the sequence $\{\mathcal{L}(X_n),\ n\ge 1\}$ is weakly relatively compact in $D[0,1]$.

Lemma 2 (cf. Remark 2. in Ref. 7). Let $\gamma_1,\gamma_2>0$. Let $\{X_n,n\ge 1\}$ be sequence of processes with sample paths in $D[0,1]$. Assume that the sequence $\{X_n,n\ge 1\}$ is equicontinuous in probability at the points $\{0;1\}$ and for each t in

$[0,1]$ the sequence of real random variables $\{X_n(t), n \geq 1\}$ is stochastically bounded. Assume that there exist a constant $c > 0$ and a number $\varepsilon > 0$ such that for each $n \geq 1$ and $\lambda > 0$,

$$\mathbf{P}(\Delta_{X_n}(s,t,u) \geq \lambda) \leq$$

$$c\lambda^{-\gamma_1}|u-s| \ln^{-\gamma_1-1-\varepsilon}(1+|u-s|^{-1}) + c\lambda^{-\gamma_2}|u-s| \ln^{-\gamma_2-1-\varepsilon}(1+|u-s|^{-1}),$$

$0 \leq s \leq t \leq u$, then the sequence $\{\mathcal{L}(X_n), n \geq 1\}$ is weakly relatively compact in $D[0,1]$.

Proof of Lemma 2. In the proof we use the scheme of (finite–dimensional) approximation elaborated in Ref. 5. By Theorem 15.3 in Ref. 6, it suffices to show that for each $\lambda > 0$

$$\sup_n \mathbf{P}(W(X_n, \eta) > \lambda) \to 0 \quad \text{as} \quad \eta \to 0, \tag{2.4}$$

where

$$W(x,\eta) := \sup\{\Delta_x(s,t,u) : \ 0 \leq s \leq t \leq u \leq 1, \ |u-s| \leq \eta\} +$$

$$\sup\{|x(t)-x(0)| + |x(1-t)-x(1)| : \ 0 \leq t \leq \eta\},$$

$x \in D[0,1]$. Denote

$$S'_k = \{j \cdot 2^{-k}, 0 \leq j \leq 2^k\}, \quad t_k^+ = \inf\{s \in S'_k, s > t\}, \quad t_k^- = \sup\{s \in S'_k, s \leq t\}.$$

By Lemma 1.3.1 in Ref. 5,

$$W(X_n, 2^{-J-1}) \leq 4I_1 + I_2 + I_3 + I_4,$$

where

$$I_1 = \sum_{k \geq J} \sup_{t \in S'_{k+1} \setminus S'_k} \Delta_{X_n}(t_k^-, t, t_k^+), \quad I_2 = \sup_{s \in S'_J \setminus \{0;1\}} \Delta_{X_n}(s - 2^{-J}, s, s + 2^{-J})$$

and

$$I_3 = \big|X_n(0) - X_n(2^{-J})\big|, \qquad I_4 = \big|X_n(1) - X_n(1-2^{-J})\big|.$$

We have

$$\mathbf{P}(W(X_n, 2^{-J-1}) > 7\lambda) \leq \sum_{j=1}^{4} \mathbf{P}(I_j > \lambda). \tag{2.5}$$

Put

$$c_0^{-1} = \sum_{k \geq 1} k^{-1} \log^{-2}(2+k).$$

Then

$$\mathbf{P}(I_1 > \lambda) \leq$$

$$\mathbf{P}(I_1 > \lambda c_0 \sum_{k \geq J} k^{-1} \log^{-2}(2+k)) \leq$$

$$\sum_{k\ge J}\mathbf{P}\big(\sup_{t\in S'_{k+1}\setminus S'_k}\Delta_{X_n}(t_k^-,t,t_k^+)>\lambda c_0 k^{-1}\log^{-2}(2+k)\big)\le$$

$$c\sum_{k\ge J}2^k\big[\big(\lambda k^{-1}\log^{-2}(2+k)\big)^{-\gamma_1}2^{-k}\log^{-1-\gamma_1-\varepsilon}(2^k)+$$

$$\big(\lambda k^{-1}\log^{-2}(2+k)\big)^{-\gamma_2}2^{-k}\log^{-1-\gamma_2-\varepsilon}(2^k)\big]\le$$

$$c\lambda^{-\gamma_1}\sum_{k\ge J}k^{-1-\varepsilon}\log^{2\gamma_1}(2+k)+c\lambda^{-\gamma_2}\sum_{k\ge J}k^{-1-\varepsilon}\log^{2\gamma_2}(2+k)=o(1) \qquad (2.6)$$

as $J\to+\infty$. Similarly,

$$\mathbf{P}(I_2>\lambda)\le c\lambda^{-\gamma_1}J^{-1-\gamma_1-\varepsilon}+c\lambda^{-\gamma_2}J^{-1-\gamma_2-\varepsilon}. \qquad (2.7)$$

Now (2.4) follows from (2.5), (2.6), (2.7) and the fact that I_3 and I_4 tend to zero in probability, since the sequence $\{X_n, n\ge 1\}$ is equicontinuous in probability at the points $\{0;1\}$. Lemma 2 is proved

Proof of theorem 1. Assume that all random variables considered in the theorem are defined on a common probability space, say $(\Omega,\mathcal{F},P)$. First we construct a version of X with sample paths in $D[0,1]$. By (1.1), (1.4), the process X is stochastically continuous. It follows from (2.2) and (1.2), (1.3) that

$$\mathbf{E}\,\Delta_X(s,t,u)\,\mathrm{I\!I}\{A\}\le c(p)\big(\Lambda_p(\Delta_X(s,t,u))\big)^{1/p}\mathbf{P}(A)^{1-1/p}$$

$$\le c(p)f^{1/p}(u-s)\mathbf{P}(A)^{1-1/p}.$$

Now an application of Theorem 1.2 of Ref. 5 yields that the process X has a version with sample paths in $D[0,1]$. Hence, in what follows we may and shall assume (without loss of generality) that X is cadlag.

It remains to prove the weak compactness of the sequence $\{\mathcal{L}(S_n),\ n\ge 1\}$. For this purpose we use Theorem 4. It follows from Lemma 3 below that

$$\mathbf{P}(\Delta_{S_n}(s,t,u)>\lambda)$$

$$\le c(p)\lambda^{-p}f(u-s)+c(p)\lambda^{-2p}(f^2(u-s)+g^2(u-s)),\quad 0\le s\le t\le u\le 1.$$

Fix $0\le s\le t\le u\le 1$. We have

$$\Delta_{S_n}(s,t,u)\le Z_1+Z_2,$$

where

$$Z_1=Z\,\mathrm{I\!I}\big\{|Z|>m\big\},\quad Z_2=Z\,\mathrm{I\!I}\big\{|Z|\le m\big\},\quad Z=\Delta_{S_n}(s,t,u)$$

and

$$m=\Big(\big(f^2(u-s)+g^2(u-s)\big)/f(u-s)\Big)^{-1/p}.$$

Simple calculation shows that

$$\Lambda_p(Z_1)\le 2c(p)\,f(u-s)\qquad\text{and}\qquad \Lambda_{2p}(Z_2)\le 2c(p)\,\big(f^2(u-s)+g^2(u-s)\big).$$

By (2.2), for each $A \in \mathcal{F}$,

$$\mathbf{E}\, Z_1\, \mathbb{I}\{A\} \le c(p) f^{1/p}(u-s)\mathbf{P}^{1-1/p}(A)$$

and

$$\begin{aligned}\mathbf{E}\, Z_2\, \mathbb{I}\{A\} &\le c(p)\big(f^2(u-s)+g^2(u-s)\big)^{1/2p}\mathbf{P}^{1-1/2p}(A)\\ &\le c(p)\big(f^{1/p}(u-s)+g^{1/p}(u-s)\big)\mathbf{P}^{1-1/2p}(A).\end{aligned}$$

Hence, for each $A \in \mathcal{F}$,

$$\mathbf{E}\Delta_{S_n}(s,t,u)\mathbb{I}\{A\} \le c(p)f^{1/p}(u-s)P(A)^{1-1/p} + c(p)g^{1/p}(u-s)P(A)^{1-1/(2p)}.$$

An application of Theorem 4 yields the weak compactness of the sequence $\{S_n,\ n \ge 1\}$. This together with the condition (i) completes the proof.

Proof of theorem 2. To construct a cadlag version X' we proceed as in the proof of Theorem 15.7 in Ref. 6. Only now we use Proposition 1 in Ref. 7 instead of Theorem 12.5 in Ref. 6 which was used in the proof of Theorem 15.7 ibidem. Hence, in what follows we may and shall assume (without loss of generality) that X is cadlag.

Lemmas 2 and 3 (below) and condition (i) yield that the sequence $\{\mathcal{L}(S_n),\ n \ge 1\}$ converges weakly to some p-stable distribution on $D[0,1]$ and complete the proof.

Lemma 3. Let $0 < p < 2$. Let X be a random process satisfying conditions (1.1) and (1.2). Let $X_1, X_2, \dots$ be independent copies of X. Assume that X is centered if $p > 1$, and that X is symmetric if $p = 1$. Then for each $\lambda > 0$ and each $n \ge 1$,

$$\begin{gathered}\mathbf{P}(\Delta_{S_n}(s,t,u) > \lambda)\\ \le c(p)\lambda^{-p} f(u-s) + c(p)\lambda^{-2p}(f^2(u-s)+g^2(u-s)),\quad 0 \le s \le t \le u \le 1.\end{gathered}$$

Proof of lemma 3. We need to estimate the probability $\mathbf{P}(\Delta_{S_n}(s,t,u) > \lambda)$. Firstly, we estimate this probability in the case when processes are symmetric.

Let $X', X_1', X_2', \dots$ be independent copies of the r. process X. Let $\varepsilon, \varepsilon_1, \varepsilon_2, \dots$ be i.i.d. Bernoulli r. variables. We assume that the sequences $\{X_i, i \ge 1\}$, $\{X_i', i \ge 1\}$ and $\{\varepsilon_i, i \ge 1\}$ are independent. Denote

$$X^* = \varepsilon(X - X'),\quad S_n' = n^{-1/p}\sum_{i=1}^n X_i',\quad S_n^* = n^{-1/p}\sum_{i=1}^n \varepsilon_i(X_i - X_i').$$

Put

$$x^* = n^{-1/p}(X^*(t) - X^*(s)),\quad y^* = n^{-1/p}(X^*(u) - X^*(t)),\quad m^* = |x^*| \wedge |y^*|$$

and

$$u = |x^*| - m^*,\quad v = |y^*| - m^*,\quad \alpha = \mathrm{sign}(x^*),\quad \beta = \mathrm{sign}(y^*).$$

We have

$$\Delta_{S_n^*}(s,t,u) =$$

$$|\sum_{i=1}^n \varepsilon_i\alpha_i m_i^* + \sum_{i=1}^n \varepsilon_i\alpha_i u_i| \wedge |\sum_{i=1}^n \varepsilon_i\beta_i m_i^* + \sum_{i=1}^n \varepsilon_i\beta_i v_i| \le$$

$$\Big|\sum_{i=1}^n \varepsilon_i\alpha_i m_i^*\Big| + \Big|\sum_{i=1}^n \varepsilon_i\beta_i m_i^*\Big| + \Big|\sum_{i=1}^n \varepsilon_i\alpha_i u_i\Big| \wedge \Big|\sum_{i=1}^n \varepsilon_i\beta_i v_i\Big|.$$

Hence,

$$\mathbf{P}(\Delta_{S_n^*}(s,t,u) \ge \lambda) \le I_1 + I_2 + I_3, \tag{2.8}$$

where

$$I_1 = \mathbf{P}\Big(\Big|\sum_{i=1}^n \varepsilon_i\alpha_i m_i^*\Big| \ge \lambda/3\Big), \quad I_2 = \mathbf{P}\Big(\Big|\sum_{i=1}^n \varepsilon_i\beta_i m_i^*\Big| \ge \lambda/3\Big)$$

and

$$I_3 = \mathbf{P}\Big(|\sum_{i=1}^n \varepsilon_i\alpha_i u_i| \wedge |\sum_{i=1}^n \varepsilon_i\beta_i v_i| \ge \lambda/3\Big).$$

Let us estimate I_1 and I_2. Put

$$x = n^{-1/p}(X(t) - X(s)), \quad y = n^{-1/p}(X(u) - X(t)), \quad x' = n^{-1/p}(X'(t) - X'(s)),$$

$$y' = n^{-1/p}(X'(u) - X'(t)), \quad m = |x| \wedge |y|, \quad m' = |x'| \wedge |y'|.$$

By Hoffmann-Jorgensen's inequality, see Lemma **1**,

$$\mathbf{P}(|\sum_{i=1}^n \varepsilon_i\alpha_i m_i^*| \ge 3\lambda) \le \sum_{i=1}^n \mathbf{P}(|m_i^*| \ge \lambda) + \mathbf{P}^2(|\sum_{i=1}^n \varepsilon_i\alpha_i m_i^*| \ge \lambda). \tag{2.9}$$

The inequality

$$|m^*| \le m + m' + |x'| \wedge |y| + |x| \wedge |y'|$$

yields

$$\mathbf{P}(m_i^* \ge 4\lambda) \le \mathbf{P}(m \ge \lambda) + \mathbf{P}(m' \ge \lambda) + \mathbf{P}(|x'| \wedge |y| \ge \lambda) + \mathbf{P}(|x| \wedge |y'| \ge \lambda).$$

Since X and X' are independent, we have

$$\mathbf{P}(m_i^* \ge \lambda) \le cn^{-1}\lambda^{-p}2f(u-s) + cn^{-2}\lambda^{-2p}2g^2(u-s) \qquad (2.10).$$

Hence, the first summand in the right-hand side of (2.9)

$$\sum_{i=1}^n \mathbf{P}(m_i^* \ge \lambda) \le c\lambda^{-p}f(u-s) + c\lambda^{-2p}g^2(u-s). \tag{2.11}$$

On the other hand,

$$\mathbf{P}(m_i^* \ge 4\lambda) \le \mathbf{P}(m \ge \lambda) + \mathbf{P}(m' \ge \lambda) + \mathbf{P}(|x'| \ge \lambda) + \mathbf{P}(|y'| \ge \lambda) \le$$

$$n^{-1}\lambda^{-p}2f(u-s) + n^{-1}\lambda^{-p}2g(u-s).$$

Therefore,

$$\Lambda_p(m_i^*) \le cn^{-1}\big(f(u-s)+g(u-s)\big) \qquad (2.12).$$

To estimate the last summand of (2.9) we combine (2.12) and (2.1):

$$\mathbf{P}(|\sum_{i=1}^n \varepsilon_i\alpha_i m_i^*| \ge \lambda) \le$$

$$\lambda^{-p}\Lambda_p\big(\sum_{i=1}^n \varepsilon_i\alpha_i m_i^*\big) \le c(p)\lambda^{-p}\sum_{i=1}^n \Lambda_p(m_i^*) \le c(p)\lambda^{-p}\big(f(u-s)+g(u-s)\big).$$

We have

$$\mathbf{P}^2\big(|\sum_{i=1}^n \varepsilon_i\alpha_i m_i^*| \ge \lambda\big) \le c(p)\lambda^{-2p}\big(f^2(u-s)+g^2(u-s)\big). \qquad (2.13)$$

Combining (2.9), (2.11), and (2.13) we get

$$\mathbf{P}\big(|\sum_{i=1}^n \varepsilon_i\alpha_i m_i^*| \ge \lambda\big) \le c\lambda^{-p}f(u-s) + c(p)\lambda^{-2p}(f^2(u-s)+g^2(u-s)).$$

In what follows we estimate the probability I_3 from (2.8). For this purpose we prove that

$$\Lambda_{2p}\Big(\big|\sum_{i=1}^n \varepsilon_i\alpha_i u_i\big| \wedge \big|\sum_{i=1}^n \varepsilon_i\beta_i u_i\big|\Big) \le c(p)\sum_{i=1}^n \Lambda_p(u_i)\,\sum_{i=1}^n \Lambda_p(v_i). \qquad (2.14)$$

It suffices to show that

$$\mathbf{P}\Big(\big|\sum_{i=1}^n \varepsilon_i\alpha_i u_i\big| \wedge \big|\sum_{i=1}^n \varepsilon_i\beta_i u_i\big| > 1\Big) \le c(p)\sum_{i=1}^n \Lambda_p(u_i)\,\sum_{i=1}^n \Lambda_p(v_i). \qquad (2.15)$$

Indeed, replacing u_i and v_i by $t^{-1}\,u_i$ and $t^{-1}\,v_i$ in (2.15) and using the identity $\Lambda_p\big(t^{-1}\,z\big) = t^{-p}\Lambda_p(z)$ with $z = u_i,\, v_i$ we obtain (2.14). Let us prove (2.15). Since in each pair $(u_i,\, v_i)$ at least one variable takes value zero, we have that $\sum_{i=1}^n \varepsilon_i\alpha_i u_i$ and $\sum_{i=1}^n \varepsilon_i\beta_i v_i$ are conditionally independent given $\alpha_i\,u_i, \beta_i\,v_i$, $1 \le i \le n$. Hence, if $\mathbf{P}_\varepsilon(A)$ denotes the conditional probability of the event A given $\alpha_i\,u_i, \beta_i\,v_i$, $1 \le i \le n$, then

$$\mathbf{P}_\varepsilon\Big(\big|\sum_{i=1}^n \varepsilon_i\alpha_i u_i\big| \wedge \big|\sum_{i=1}^n \varepsilon_i\beta_i u_i\big| > 1\Big) = \mathbf{P}_\varepsilon\Big(\big|\sum_{i=1}^n \varepsilon_i\alpha_i u_i\big| > 1\Big)\mathbf{P}_\varepsilon\Big(\big|\sum_{i=1}^n \varepsilon_i\beta_i u_i\big| > 1\Big). \qquad (2.16)$$

Furthermore, we have

$$\mathbf{P}_\varepsilon\Big(\big|\sum_{i=1}^n \varepsilon_i\alpha_i u_i\big| > 1\Big) \le \mathbf{P}_\varepsilon\Big(\big|\sum_{i=1}^n Y_i\big| > 1\Big) + \mathbb{I}\big\{\max_i |u_i| > 1\big\}$$

and

$$\mathbf{P}_\varepsilon\Big(\big|\sum_{i=1}^n \varepsilon_i\beta_i u_i\big| > 1\Big) \le \mathbf{P}_\varepsilon\Big(\big|\sum_{i=1}^n Z_i\big| > 1\Big) + \mathrm{I\!I}\big\{\max_i |v_i| > 1\big\},$$

where

$$Y_i = \varepsilon_i\alpha_i u_i\, \mathrm{I\!I}\big\{|u_i| \le 1\big\}, \quad Z_i = \varepsilon_i\alpha_i z_i\, \mathrm{I\!I}\big\{|z_i| \le 1\big\}, \quad i \ge 1.$$

Thus,

$$\mathbf{P}_\varepsilon\Big(\big|\sum_{i=1}^n \varepsilon_i\alpha_i u_i\big| > 1\Big)\mathbf{P}_\varepsilon\Big(\big|\sum_{i=1}^n \varepsilon_i\beta_i u_i\big| > 1\Big) = J_1 + J_2 + J_3 + J_4. \tag{2.17}$$

Here

$$J_1 = \mathbf{P}_\varepsilon\Big(\big|\sum_{i=1}^n Y_i\big| > 1\Big)\mathbf{P}_\varepsilon\Big(\big|\sum_{i=1}^n Z_i\big| > 1\Big),$$

$$J_2 = \mathrm{I\!I}\big\{\max_i |u_i| > 1\big\}\mathbf{P}_\varepsilon\Big(\big|\sum_{i=1}^n Z_i\big| > 1\Big),$$

$$J_3 = \mathbf{P}_\varepsilon\Big(\big|\sum_{i=1}^n Y_i\big| > 1\Big)\mathrm{I\!I}\big\{\max_i |v_i| > 1\big\}$$

and

$$J_4 = \mathrm{I\!I}\big\{\max_i |u_i| > 1\big\}\mathrm{I\!I}\big\{\max_i |v_i| > 1\big\}.$$

Let us estimate $\mathbf{E}\, J_1$, $\mathbf{E}\, J_2$, $\mathbf{E}\, J_3$ and $\mathbf{E}\, J_4$. Since at least one of u_i and v_i takes value zero, we have

$$J_4 \le \sum_{i \ne j} \mathrm{I\!I}\big\{|u_i| > 1\big\}\mathrm{I\!I}\big\{|v_j| > 1\big\}$$

and

$$\mathbf{E}\, J_4 \le \sum_{i \ne j} \Lambda_p(u_i)\Lambda_p(v_j). \tag{2.18}$$

Furthermore,

$$J_3 \le \sum_{i=1}^n \mathrm{I\!I}\big\{|v_i| > 1\big\}\mathbf{P}_\varepsilon\Big(\big|\sum_{1 \le j \le n,\, j \ne i} Y_j\big| > 1/2\Big) + \sum_{i=1}^n \mathrm{I\!I}\big\{|v_i| > 1\big\}\mathbf{P}_\varepsilon\big(|Y_i| > 1/2\big).$$

Observe that the last summand equals to zero. Hence,

$$\mathbf{E}\, J_3 \le \sum_{i=1}^n \Lambda_p(v_i)\mathbf{E}\,\mathbf{P}_\varepsilon\Big(\big|\sum_{1 \le j \le n,\, j \ne i} Y_j\big| > 1/2\Big) \le c(p) \sum_{i \ne j} \Lambda_p(v_i)\, \Lambda_p(u_j). \tag{2.19}$$

Here we use the estimate

$$\mathbf{E}\,\mathbf{P}_\varepsilon\Big(\big|\sum_{1 \le j \le n,\, j \ne i} Y_j\big| > 1/2\Big) = \mathbf{P}\Big(\big|\sum_{1 \le j \le n,\, j \ne i} Y_j\big| > 1/2\Big) \le c(p) \sum_{1 \le j \le n,\, j \ne i} \Lambda_p(u_j),$$

see proof of Theorem 1 in Ref. 25. The same bound holds for J_2. Let $p' \in (p;2)$ be a parameter which depends only on p. An application of Chebyshev's inequality gives

$$J_1 \le \mathbf{E}_\varepsilon \Big|\sum_{i=1}^n Y_i\Big|^{p'} \mathbf{E}_\varepsilon \Big|\sum_{j=1}^n Z_j\Big|^{p'} \le \sum_{i\neq j} \mathbf{E}_\varepsilon \big|Y_i\big|^{p'} \mathbf{E}_\varepsilon \big|Z_j\big|^{p'}.$$

Here $\mathbf{E}_\varepsilon$ denotes the expectation with respect to the r. variables $\{\varepsilon_i\}$, i.e. the conditional expectation given $\alpha_i\, u_i$ and $\beta_i\, v_i$, $1 \le i \le n$. Using estimates

$$\mathbf{E}\,\big|Y_i\big|^{p'} \le c(p)\Lambda_p(u_i) \qquad \text{and} \qquad \mathbf{E}\,\big|Z_i\big|^{p'} \le c(p)\Lambda_p(v_i),$$

see proof of Theorem 1 in Ref. 25, we obtain

$$\mathbf{E}\,J_1 \le c(p) \sum_{i\neq j} \Lambda_p(u_i)\,\Lambda_p(v_j).$$

This inequality together with (2.16-19) yields (2.15). We arrive to (2.14). Now the inequality

$$I_3 \le c(p) g^2(u-s),$$

follows from (2.14), since

$$\Lambda_p(u_i) \le c(p)\Lambda_p\big(X(s)-X(t)\big) \le g(t-s) \le g(u-s)$$

and

$$\Lambda_p(v_i) \le c(p)\Lambda_p\big(X(t)-X(u)\big) \le g(u-t) \le g(u-s).$$

By (2.8),

$$\mathbf{P}(\Delta_{S_n^*}(s,t,u) \ge \lambda) \le c(p)\lambda^{-p} f(u-s) + c(p)\lambda^{-2p}(f^2(u-s)+g^2(u-s)). \quad (2.20)$$

We will complete the proof of the lemma by showing that the same bound holds for $\mathbf{P}(\Delta_{S_n}(s,t,u) \ge \lambda)$. Denote

$$x = S_n(t)-S_n(s),\;\; y = S_n(u)-S_n(t),\;\; x' = S_n'(t)-S_n'(s),\;\; y' = S_n'(u)-S_n'(t).$$

The inequality

$$|x| \wedge |y| \le |x-x'| \wedge |y-y'| + |x'| \wedge |y| + |x| \wedge |y'|$$

implies

$$\begin{aligned} &\mathbf{P}(\Delta_{S_n}(s,t,u) > 3\lambda) \le \\ &\mathbf{P}(\Delta_{S_n^*}(s,t,u) > \lambda) + \mathbf{P}(|S_n'(t)-S_n'(s)| \wedge |S_n(u)-S_n(t)| > \lambda) \\ &\qquad + \mathbf{P}(|S_n(t)-S_n(s)| \wedge |S_n'(u)-S_n'(t)| > \lambda). \qquad (2.21) \end{aligned}$$

It follows from (2.1) and Remark 3 that

$$\mathbf{P}(|S_n(t)-S_n(s)| > \lambda)$$

$$\leq \lambda^{-p}\Lambda_p(S_n(t)-S_n(s)) \leq c(p)\lambda^{-p}\Lambda_p(X(t)-X(s)) \leq c(p)\lambda^{-p}g(t-s).$$

By the independence of S_n and S_n', the last two probabilities in (2.21) do not exceed

$$c\lambda^{-2p}g^2(u-s). \tag{2.22}$$

The lemma follows from (2.20-22).

Proof of the Proposition 1. The proposition is a consequence of Lemmas 2 and 3.

Proof of the statements of Example 2. For $\omega \equiv 1$, we have

$$\Lambda_p\big(|X(s)-X(t)| \wedge |X(t)-X(u)|\big) = 0$$

and

$$\Lambda_p\big(|X(s)-X(u)|\big) \leq \Lambda_p(V)\,\mathbf{E}\left|\mathrm{I\!I}\left\{U \in (s;\, u)\right\}\right|^p = c(V)\,|u-s|.$$

Theorems 1 and 2 imply the weak convergence of $\mathcal{L}(F_n)$, for $p \in (0;\, 2)$, $p \neq 1$.

Let $1 < p < 2$. Denote

$$Y(t) = \omega(t)\,\mathrm{I\!I}\big\{U \leq t\big\}.$$

We have

$$\Lambda_p\big(|X(s)-X(t)| \wedge |X(t)-X(u)|\big) \leq \Lambda_p(V)\,\mathbf{E}\left(|Y(s)-Y(t)| \wedge |Y(t)-Y(u)|\right)^p,$$

$$\Lambda_p\big(|X(s)-X(u)|\big) \leq \Lambda_p(V)\,\mathbf{E}\left|Y(s)-Y(u)\right|^p.$$

The weak convergence of $\mathcal{L}(F_n)$ would follow from Theorem 1 and Remark 2 if we show that for some $\varepsilon > 0$ and some continuous increasing function $H : [0;\, 1] \to R$,

$$\mathbf{E}\left(|Y(s)-Y(t)| \wedge |Y(t)-Y(u)|\right)^p \leq \left|H(u)-H(s)\right|^{1+\varepsilon} \tag{2.23}$$

and

$$\mathbf{E}\left|Y(s)-Y(u)\right|^p \leq \left|H(u)-H(s)\right|. \tag{2.24}$$

We shall choose ε such that $1 < 1+\varepsilon < \min\{p,\, \delta p\}$.

Let us construct the function H. We have

$$\mathbf{E}\left(|Y(s)-Y(t)| \wedge |Y(t)-Y(u)|\right)^p \leq I_1 + I_2,$$

where

$$I_1 = s\Big(\big|\omega(s)-\omega(t)\big| \wedge \big|\omega(t)-\omega(u)\big|\Big)^p, \quad I_2 = (t-s)\Big(\omega(t) \wedge \big|\omega(t)-\omega(u)\big|\Big)^p.$$

Fix $\gamma = e^{-e}$. Let $t, u \in (\gamma;\, 1]$. Put

$$R_1(t) := m(t) + t \qquad \text{and} \qquad M = \max_{\gamma \leq t \leq 1} m(t).$$

We have

$$\omega(u)-\omega(t) = u^{-1/p}\Big(m(u)-m(t)\Big) + m(t)\Big(u^{-1/p}-t^{-1/p}\Big)$$

$$\le \Big(\gamma^{-1/p} + M\,\gamma^{-1-1/p}\,p^{-1}\Big)\,\big(R_1(u) - R_1(t)\big).$$

Therefore,

$$I_1 + I_2 \le \big|\omega(u) - \omega(t)\big|^p \le c(p,m)\,\big(R_1(u) - R_1(t)\big)^p. \tag{2.25}$$

For $t \le \gamma$, we shall estimate I_1 and I_2 separately. Let us consider

$$I_1 \le s\big|\omega(s) - \omega(t)\big|^p = \Big|m(s) - m(t) + m(t)(t^{1/p} - s^{1/p})t^{-1/p}\Big|^p$$

$$\le 2^p\,\big|m(s) - m(t)\big|^p + I_3,$$

where

$$I_3 = 2^p\,m^p(t)\Big|(t^{1/p} - s^{1/p})t^{-1/p}\Big|^p.$$

Calculation shows that for $t^2 \le s \le t/2$, the function

$$R_2(t) := \ln_2^{-1}(t^{-1}), \quad 0 < t \le \gamma, \quad R_2(0) := 0, \quad R_2(t) := 1, \quad \gamma \le t \le 1$$

satisfies

$$R_2(t) - R_2(s) \ge c\,\ln^{-1}(t^{-1})\,\ln_2^{-2}(t^{-1}).$$

Hence,

$$I_3 \le 2^p\,m^p(t) \le c(p,m)\ln^{-\delta p}(t^{-1}) \le c(p,\varepsilon,m)\big(R_2(t) - R_2(s)\big)^{1+\varepsilon}.$$

If $t/2 \le s \le t$ then

$$R_2(t) - R_2(s) \ge c\,(t-s)t^{-1}\ln^{-1}(t^{-1})\ln_2^{-1}(t^{-1}).$$

In this case

$$I_3 \le 2^p\,m^p(t)\,c(p)\big((t-s)/t\big)^p \le c(p,\varepsilon,m)\big(R_2(t) - R_2(s)\big)^{1+\varepsilon}.$$

Finally, for $0 \le s \le t^2$,

$$I_3 \le 2^p\,m^p(t) \le c(p)\ln^{\delta p}(t^{-1}) \le c(p,\varepsilon,m)\ln^{-1-\varepsilon}(t^{-1}).$$

Obviously,

$$I_3 \le c(p,\varepsilon,m)\big(R_3(t) - R_3(s)\big)^{1+\varepsilon},$$

where

$$R_3(t) := \ln^{-1}(t^{-1}), \quad 0 < t \le \gamma, \quad R_3(0) := 0, \quad R_3(t) := e^{-1}, \quad \gamma \le t \le 1.$$

Put

$$R_4(t) := c(p,\varepsilon,m)\big(R_1(t) + R_2(t) + R_3(t)\big),$$

where $c(p,\varepsilon,m)$ is sufficiently large constant. Then

$$I_1 \le \big(R_4(t) - R_4(s)\big)^{1+\varepsilon}, \qquad 0 \le s \le t \le \gamma. \tag{2.26}$$

Let us estimate I_2. If $t \le u \le \gamma$ then proceeding as in proof of (2.26) we obtain

$$I_2 \le t\,\big|\omega(t) - \omega(u)\big|^p \le \big(R_4(u) - R_4(t)\big)^{1+\varepsilon}. \tag{2.27}$$

In the case $t \le \gamma \le u$ we have

$$I_2 \le 2^p\, t\,\big|\omega(t) - \omega(\gamma)\big|^p + 2^p\,\big|\omega(u) - \omega(\gamma)\big|^p$$

$$\le c(p)\big(R_4(t) - R_4(\gamma)\big)^{1+\varepsilon} + c(p)\big(R_1(u) - R_1(\gamma)\big)^p$$

$$\le c(p,\varepsilon)\big(R_4(u) - R_4(t)\big)^{1+\varepsilon}. \tag{2.28}$$

Combining (2.25) and (2.26-28) we obtain (2.23) with

$$H(t) := c(p,\varepsilon,m)R_4(t).$$

Let us show (2.24). We have

$$\mathbf{E}\,\big|Y(s) - Y(t)\big|^p = s\,\big|\omega(s) - \omega(t)\big|^p + (t-s)\omega^p(t).$$

The first summand is already estimated (see the estimation of I_3 above) and does not exceed

$$c(p,\varepsilon,m)\Big(R_4(t) - R_4(s)\Big)^{1+\varepsilon}.$$

Let us estimate the second summand

$$I_4 = (t-s)\omega^p(t) = (t-s)t^{-1}m^p(t).$$

For $t \ge \gamma$,

$$I_4 \le M^p\gamma^{-1}(t-s) \le c(p,m)\big(R_1(t) - R_1(s)\big).$$

To estimate I_4 for $t \le \gamma$, we proceed in the same way as that of the estimation of I_3. We obtain

$$I_4 \le c(p,m)\Big(R_4(t) - R_4(s)\Big), \qquad 0 \le t \le \gamma.$$

Therefore, (2.24) follows and this completes the proof of the statements of Example 2.

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