Edge Detection For Image Processing Using Second Directional Derivative

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ABSTRACT

One function of data preprocessing is to convert a visual pattern into an electrical pattern or to convert a set of discrete data into a mathematical pattern so that those data are more suitable for computer analysis.[1] With regard to edge detection, we define an edge to occur in a pixel if and only if there is some point in the pixel's area having a negatively sloped zero crossing of the second directional derivative which is taken in the direction of a nonzero gradient at the pixel should be marked as a step edge pixel, its underlying gray tone intensity surface(proper thresholding) must be estimated on the basis of the pixels in its neighborhood[2]. Therefore, a functional from consisting of a linear combination of the tensor products of discrete directional derivatives are easily computed form this kind of function. Various results of computer analysis by using different window size and intensity function are included in this report. The best combination of the parameters for second directional derivative in edge finding will also be presented in this study.

Key word : Intensity Function, second Directional Derivative, Zero Crossing, Orthogonal Polynomial

1.Introduction

In this report, we assume that in each neighborhood of the image the underlying gray tone intensity function 'f' takes the parametric form of a polynomial in the row and column coordinates, and that the producting digital picture function is a regular equal interval grid samppling of the square plane which is the domain of "f".Thus in each neighborhood "f"take the form of cubic surface as[3]

$$f(r,c)=K_1+K_2r+K_3c+K_4r^2+K_5rc+k_6c^2+K_7r^3+K_8r^2c+ K_9rc^2+K_{10}c^3 \tag{1}$$

where f(r,c) is the intensity function at position(r,c) along the gradient $\vec{G}$. A pixel is marked as an edge pixel if in the pixel's immediate area there is a zero crossing of the second directional derivative taken in the direction of the gradient and the slope of the zero crossing is negative. Having used the pixel values in a neighborhood to estimate the underlying polynomial function we can then determine the value of the partial derivatives at any location in the neighborhood and use those values in edge finding.

2.Fitting Data with Discrete Orthogonal polynomials

Let an index set "R" with the symmetry property, Thus for two dimension, if we let the number of elements in "R" is N $*$ N and using the construction technique of orthogonal properties, we may come up the equation as

$$\mathbf{d(r,c)} = \sum \mathbf{a_n p_n}(\mathbf{r},\mathbf{c}) + \mathrm{Error}(\mathbf{r},\mathbf{c}) \tag{2}$$

using the tensor product technique we can construct discrete orthogonal polynomials over a two-dimensional neighborhood for 4 $*$ 4 elements as :[4]

Index set:

$$(-1.5,-0.5,0.5,1.5) * (-1.5,-0.5,0.5,1.5) \tag{3}$$

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Discrete orthogonal polynomial set:

$$(1,r,r^2-\frac{5}{4},r^3-\frac{41}{20}r) * (1,c,c^2-\frac{5}{4},c^3-\frac{41}{20}c) \tag{4}$$

For each(r,c) in "R", let a data value d(r,c) be observed. The exact fitting problem is to determine coefficients, i.e.

$$a_m = \sum P_m(r,c)d(r,c) / \sum P_m(r,c) * P_m(r,c) \tag{5}$$

where m=0,1,2 ....N * N-1. The detailed mathematics operation and results are given as follows: [5]

$$(1,r,r^2-\frac{u_2}{u_0},r^3-\frac{u_4}{u_2}r) \times (1,c,c^2-\frac{u_2}{u_0},c^3-\frac{u_4}{u_2}c) \tag{6}$$

3.Theoretical Analysis

By combining the expression of equation (6). and equation (5), obtained the following coefficients :

$$a_0 = \sum 1 / \sum 1 \times 1$$
$$a_1 = \sum c \times d(r,c) / \sum c \times c$$
$$a_2 = \sum (c^2-\frac{u_2}{u_0})d(r,c) / \sum (c^2-\frac{u_2}{u_0})(c^2-\frac{u_2}{u_0})$$
$$a_3 = \sum (c^3-\frac{u^4}{u^2})d(r,c) / \sum (c^3-\frac{u_4}{u_2}c)(c^3-\frac{u_4}{u_2}c)$$
$$a_4 = \sum r \times d(r,c) / \sum r \times r$$
$$a_5 = \sum rc \times d(r,c) / \sum rc \times rc$$

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$$a_{15} = \frac{\sum (r^3-\frac{u_4}{u_2})(c^3-\frac{u_4}{u_2}c) \times d(r,c)}{\sum [(r^3-\frac{u_4}{u_2}r)(c^3-\frac{u_4}{u_2}c)]^2} \tag{7}$$

From Nalwa Binford technique [6], K is chosen so that the sum of the elements in the mask "C" can be used in integer arithmetic, Thus,

$$K_2 = a_4 - a_6\frac{u_2}{u_0} - a_{12}\frac{u_4}{u_2} + a_{14}\frac{u_2}{u_0} \times \frac{u_4}{u_2} \tag{8}$$

$$K_3 = a_1 - a_3\frac{u_4}{u_2} - a_9\frac{u_2}{u_0} + a_{11}\frac{u_2}{u_0} \times \frac{u_4}{u_2} \tag{9}$$

$$K_4 = a_8 - a_{10}\frac{u_2}{u_0} \tag{10}$$

$$K_5 = a_5 - a_7\frac{u_4}{u_2} - a_{13}\frac{u_4}{u_2} + a_{15}\frac{u_4}{u_2} \times \frac{u_4}{u_2} \tag{11}$$

$$K_6 = a_2 - a_{10}\frac{u_2}{u_0} \tag{12}$$

$$K_7 = a_{12} - a_{14}\frac{u_2}{u_0} \tag{13}$$

$$K_8 = a_9 - a_{11}\frac{u_4}{u_2} \tag{14}$$

$$K_9 = a_6 - a_{14}\frac{u_4}{u_2} \tag{15}$$

$$K_{10} = a_3 - a_{11}\frac{u_2}{u_0} \tag{16}$$

The derivatives of the intensity function are:[7]

$$\begin{aligned} f'\alpha(r,c) &= \frac{\partial}{\partial r}f(r,c)\sin\alpha + \frac{\partial}{\partial c}f(r,c)\cos\alpha \\ &= (K_2\sin\alpha + K_3\cos\alpha) + 2(K_4\sin^2\alpha + K_5\sin\alpha\cos\alpha \\ &\quad + K_6\cos^2\alpha)\rho + 3(K_7\sin^3\alpha\cos\alpha + K_8\sin^2\alpha\cos\alpha \\ &\quad + K_9\sin\alpha\cos^2\alpha + K_{10}\cos^3\alpha)\rho^2 \end{aligned} \tag{17}$$

Therefore,

$$\begin{aligned} f''\alpha(r,c) &= 2(k_4\sin^2\alpha + K_5\sin\alpha\cos\alpha + K_6\cos^2\alpha) + \\ &\quad 6(K_7\sin^3\alpha + K_8\sin^2\alpha\cos\alpha + K_9\sin\alpha\cos^2\alpha \\ &\quad + K_{10}\cos^3\alpha)\rho \end{aligned} \tag{18}$$

$$\begin{aligned} f'''\alpha(r,c) &= 6(K_7\sin^3\alpha + K_8\sin^2\alpha\cos\alpha + K_9\sin\alpha\cos^2\alpha \\ &\quad + K_{10}\cos^3\alpha) \end{aligned} \tag{19}$$

and

$$\sin\alpha = K_2 / \sqrt{K_2^2 + K_3^2} \tag{20}$$

$$\cos\alpha = K_3 / \sqrt{K_2^2 + K_3^2} \tag{21}$$

$$r = \rho\sin\alpha, c = \rho\cos\alpha \tag{22}$$

where ρ is the distance of the point along the gradient from center pixel. If in a small neighborhood, one has $f'''(\rho) < 0, f''(\rho) = 0$ and $f'(\rho) \neq 0$ then mark the center pixel as an edge pixel.

4.Result and Conclusion

The actual image and backgrown part were shown in Fig.1and Fig.2. By choosing the different parameters (f', f'', f''') and the window size, the result of computer analysis were shown from Fig.3 to Fig.8 . If for some $\rho, |\rho| < \rho'$, where ρ' is slightly smaller than the length of the side of a pixel,i.e.$\rho''' \alpha(\rho) < 0$ and $f''\alpha(\rho) = 0$ and $f'\alpha(\rho) \neq 0$, we have discovered a negatively sloped zero crossing of the estimated second directional derivative taken in the estimated direction of the gradient and we mark the center pixel of the neighborhood as an edge pixel.After a number of testing, we obtain the following conclusions.

(1)If the gray level near the edge pixel have no much difference in value, such kind of edge is difficult to detect. (see Fig.3.)

(2)It is important to use larger neighborhood size than 3 * 3 and have shown that better results are achieved by defining the edge operator naturally in the large neighborhood 4 * 4 rather than preaveraging and then using a smaller neighborhood edge operator on the averaged image.(see Fig.5and Fig.6)

(3)The 1st and 2nd derivatives of intensity function, have a significant effect for the edge detection when its absolute value is changed. (refer Fig3 to Fig6.)

(4)When window size larger than 4 * 4,the edge detection for image processing has a negative influence due to the expansion of resolution (refer Fig.6 and Fig.8.)

(5)By considering the proper window size and the boundary value of the intensity function, the best result for the binary image in edge finding is given in a Fig.6.

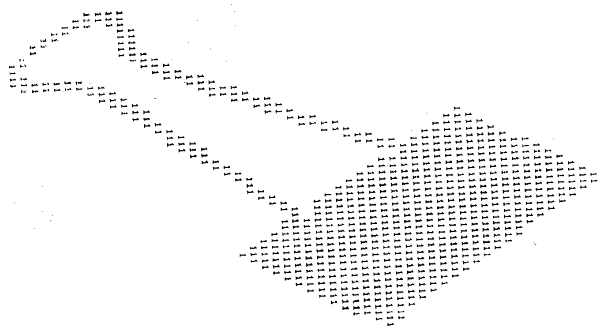

Fig.1 Binary Image (Actual Part)

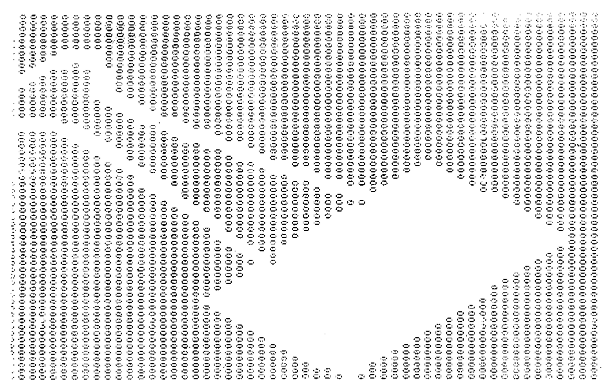

Fig.2 Binary Image (Background Part)

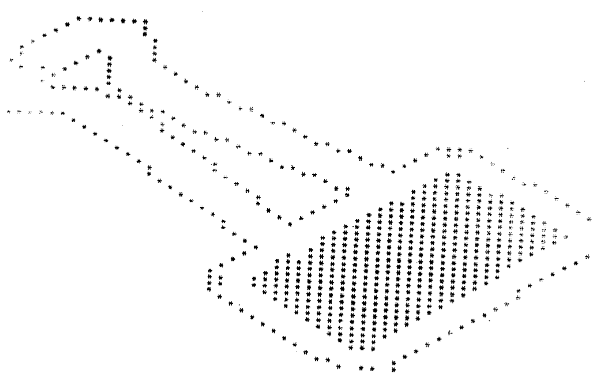

Fig.3 Edge Detection For Binary Image
Window Size (4 * 4)
$f''' < 0$, $f' < 0.25$, $f' > 0$

5.Reference

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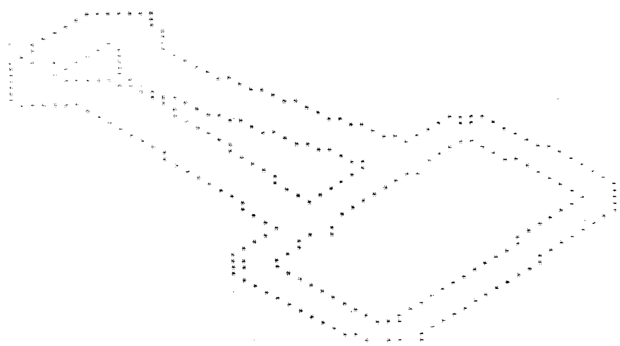

Fig. 4 Edge Detection For Binary Image
$f''' < 0$, $f'' < 0.25$, $f' > 0.001$
Window Size (4 * 4)

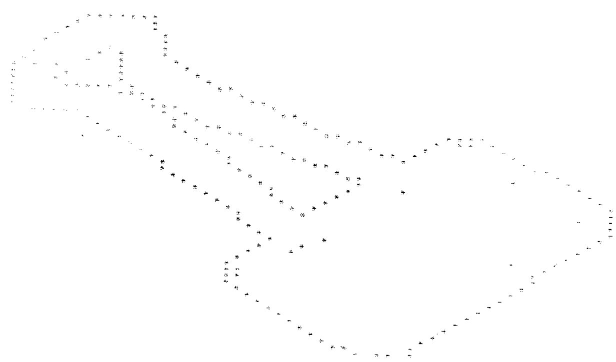

Fig. 5 Edge Detection For Binary Image
$f''' < 0$, $f'' < 0.25$, $f' > 0.005$
Window Size (4 * 4)

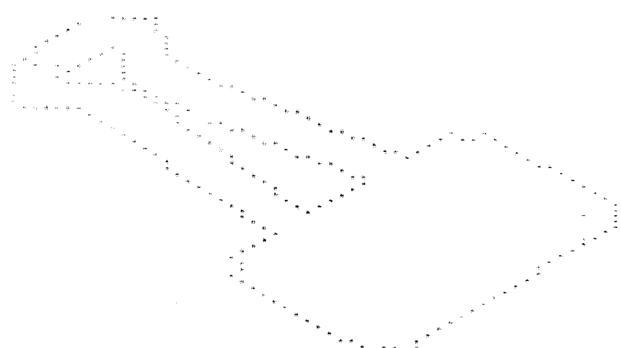

Fig.6 Edge Detection For Binary Image
$f''' < 0$, $f'' < 0.25$, $f' > 0.013$
Window Size (4 * 4)

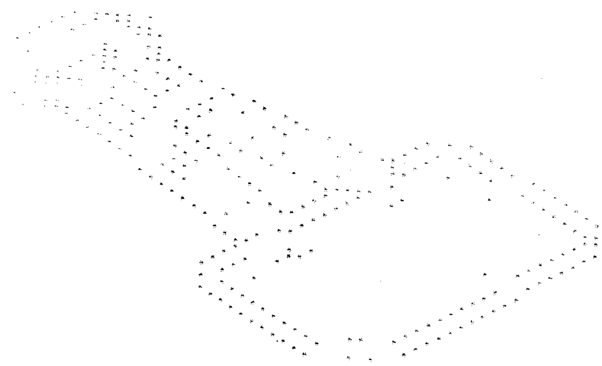

Fig. 7 Edge Detection For Binary Image
$f''' < 0$, $f'' < 0.25$, $f' > 0.013$
Window Size (5 * 5)

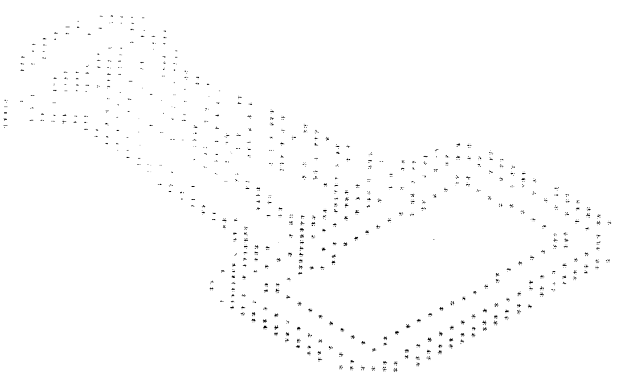

Fig. 8 Edge Detection For Binary Image
$f''' < 0$, $f'' < 0.25$, $f' > 0.013$
Window Size (6 * 6)