# Image Enhancement in Real Space

The goal of this lecture is to show how modifications to the image may be made, for the purpose of enhancement through transformations applied to the individual gray levels of the image: a process sometimes referred to as "gray level transformation".

HISTOGRAM PROCESSING

A histogram of the frequency, or number of times, that a pixel with a particular gray-level occurs within an image provides us with a useful statistical representation of the image.

As an example, consider the image below which represents a dark object (a square on light background. The object is represented by gray levels greater than 4, and the background by levels between 0-2.

| 0	0	1	0	1	0	2	0
1	0	0	1	0	2	0	1
0	1	7	5	6	6	0	1
0	0	6	5	5	7	2	0
1	1	7	7	7	6	0	0
0	2	6	6	6	5	2	1
1	1	2	1	1	0	0	0
1	0	2	2	1	0	1	2
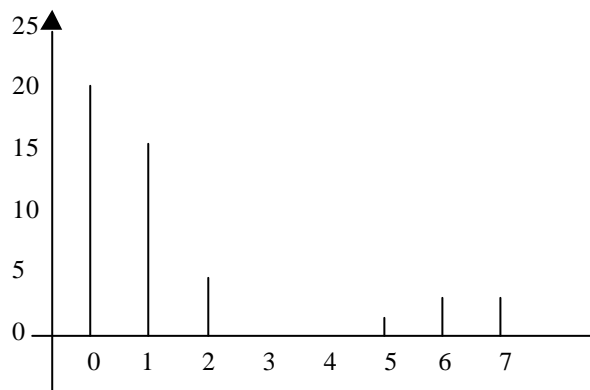
The above histogram is referred to as being 'bimodal' as it consists of two peaks: one in the region spanning gray levels 0-2 and the other spanning gray levels 5-7.

Histograms only showing one significant peak are called unimodal.

In the case of complex images eg satellite or medical images which may consist of up to 256 gray levels and 3000*3000 pixels, the resulting histograms will consist of many peaks and the distribution of those peaks together with the magnitude, can reveal significant information about the information content of the image.

Ideally, we would like an image whose histograms shows a ***uniform distribution*** over the range of gray levels covered, since this enables us to ***comprehend*** the contribution made by all the gray levels to the image.

In practice, however, the resulting histogram is often nonuniform, with a few levels dominating over others to the extent that full image interpretation by the eye ***cannot*** be achieved.
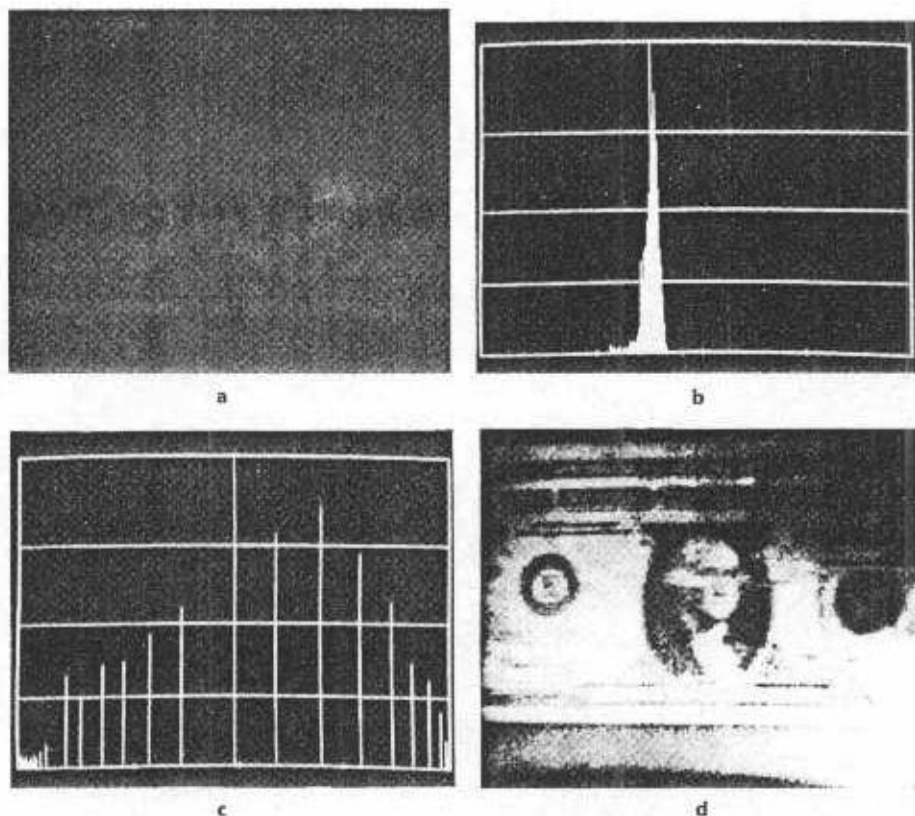
We therefore look for methods which transform the available intensities over a broader range. Figure (a) for example shows an image and its corresponding histogram is shown in figure (b). The horizontal axis in (b) represents a gray scale range from 0-255 and the vertical axis represents the frequency of occurrence of

each gray level in the range. Note the "bunching" of gray level values in the lower part of the range which corresponds to the rather dark appearance of the image.

Figure (d) shows the gray level distribution after the application of a transformation, which effectively enhances the scale of values relative to the original scale. The validity of this approach to enhance the scale is based upon statistical theory and is explained in more detail later.

Figure (c) shows the new image resulting from the gray level redistribution process. This process has increase the **dynamic range** (the ratio between the minimum and maximum of intensities) of intensities to include more gray scale, with the result that the overall contrast is enhanced relative to the original image. This transformation process is called "histogram equalisation" and the following section explains the method of implementation.

HISTOGRAM EQUALISATION

Ideally we would like our image histogram to be distributed across the range of gray scale values as a **uniform** distribution. This means that our contrast will be at an optimal level for subsequent image interpretation since **all levels are represented**. This is not generally the case in practice however and the distribution may be dominated by a few values spanning only a limited range.

Statistical theory shows that by using a transformation function equal to the cumulative distribution of the gray level intensities in the image, enables us to generate another image with a gray level distribution having a uniform density.

The implementation of the above transformation can be summarised as a three stage process:

1. obtain a histogram of the image;
2. obtain the cumulative distribution of the gray levels;
3. replace the original gray level intensities by those determined in (2).

To illustrate the steps above we refer to the image defined by matrix which represents a square on a light background.

The gray scale distribution of the individual levels is denoted by g_k where $k = 0, 1, \ldots, 7$ and is normalised by dividing the range by the maximum level, in this case 7.

The frequency of occurrence of a particular gray level is denoted by f_k and S_k represents the cumulative sum which may be defined as $S_k = f_r/n$ with n representing the total number of pixels in the image:

g_k	f_k/n	S_k
0/7	22/64	22/64
1/7	17/64	39/64
2/7	9/64	48/64
3/7	0/64	48/64
4/7	0/64	48/64
5/7	4/64	52/64
6/7	7/64	59/64
7/7	5/64	64/64

Our S_k array now represents a new set of gray level values to replace the original values ie *0/7* above is to be replaced with *22/64*. Similarly, the value *1/7* is replaced by the value *39/64* etc.

However, since our range of allowable levels is from 0-7 (this may be the only range available to the display facilities), we need to equate our new levels to be compatible within this range. This done by simply rewriting both the g_k and S_k values in decimal form and then select those values from g_k closest to those in S_k:

g_k	S_k
0.00	0.34
0.14	0.61
0.28	0.75
0.43	0.75
0.57	0.75
0.71	0.81
0.86	0.92
1.00	1.00

The closest value to S_0 ie *0.34* is g_2 ie *0.28* which in turn corresponds to the gray level value *2/7*. Similarly, the closest value to S_1 is g_4 which corresponds to the gray level value of *4/7*. Continuation of the process yields the corresponding gray level values for the set S_k as:

Sk = (2/7, 4/7, 5/7, 5/7, 5/7, 6/7, 6/7, 7/7) for k = 0, 1, …, 7

Substituting these values for those in the original image yields a new image as shown below:

2	2	4	2	4	2	5	2
4	2	2	4	2	5	2	4
2	4	7	6	6	6	2	4
2	2	6	6	6	7	5	2
4	4	7	7	7	6	2	2
2	5	6	6	6	6	5	4
4	4	5	4	4	2	2	2
4	2	5	5	4	2	4	5

It should be emphasised that the above image is the result of applying the method to a frequency distribution in which the available levels were reasonably distributed and is only presented for the purposes of illustrating the method.

The effectiveness of the approach is best appreciated by reference to picture above which represents an image of the type more likely to be found in practice.

HISTOGRAM SPECIFICATION

Although the above method enables the redistribution of gray levels over a uniform range, circumstances may arise where certain specific levels need to be highlighted relative to others. For example, even after application of the equalisation process, certain levels may still dominate the image to the extent that the contribution of the other levels *cannot be interpreted* by the eye.

One way to overcome this is to specify a histogram distribution which enhances selected gray levels relative to others and then reconstitute the original image in terms of the new distribution.

The implementation of the method can be illustrated in a similar way to that shown for the case of equalisation and consists of the three basic steps below.

1. apply the process of equalisation to the image;
2. specify a gray level distribution which enhances certain levels relative to others;
3. determine the compatible relationship between the levels determined in (1) with those determined in (2).

With reference to matrix (figure above) we repeat the results of the equalisation process showing the gray levels distributions in both fractional and decimal form:

g_k	f_k/n	S_k
0.7/ (0.00)	22/64	22/64 (0.34.)
1/7 (0.14)	17/64	39/64 (0.61)
2/7 (0.28)	9/64	48/64 (0.75)
3/7 (0.43)	0/64	48/64 (0.75)
4/7 (0.57)	0/64	48/64 (0.75)
5/7 (0.71)	4/64	52/64 (0.81)
6/7 (0.86)	7/64	59/64 (0.92)
7/7 (1.00)	5/64	64/64 (1.00)

In accordance with step 2 above we now specify a distribution with the appropriate properties. For example, we may decide to **reduce** the levels between 0 and 2 ie the background levels and **increase** the levels between 5 and 7 correspondingly. Suppose we further decide that the amount by which we choose to both correspondingly reduce and increase the frequency distribution is arbitrarily set as **7/64**.

Then our new distributions may be written as:

g'_k	f'_k/n	S'_n
0/7 (0.00)	15/64	15/64 (0.23)
1/7 (0.14)	10/64	25/64 (0.39)
2/7 (0.28)	2/64	27/64 (0.42)
3/7 (0.43)	0/64	27/64 (0.42)
4/7 (0.57)	0/64	27/64 (0.42)
5/7 (0.71)	11/64	38/64 (0.59)
6/7 (0.86)	14/64	52/64 (0.81)

7/7 (1.00)	12/64	64/64 (1.00)

In accordance with step (3) above, we now equate the level S_k to those of S'_k as follows. The closest match to S_0 ie **0.34** is S'_1, ie **0.39** which corresponds to level **1/7**. Similarly, the closest match to S_1, ie **0.61** is S'_5 ie **0.59** which corresponds to level **5/7**. Continuation of the process gives the following set of mappings:

S_0	$g'_1 = 1/7$
S_1	$g'_5 = 5/7$
S_2	$g'_6 = 6/7$
S_3	$g'_6 = 6/7$
S_4	$g'_6 = 6/7$
S_5	$g'_6 = 6/7$
S_6	$g'_7 = 7/7$
S_7	$g'_7 = 7/7$

Insertion of these levels in place of those comprising the histogram equalised image give the new image shown below:

| 1	1	5	1	5	1	6	1
5	1	1	5	1	6	1	5
1	5	7	7	7	7	1	5
1	1	7	7	7	7	6	1
5	5	7	7	7	7	6	1
1	6	7	7	7	7	1	1
5	5	6	5	5	1	1	1
5	1	6	6	5	1	5	6

To illustrate the effectiveness of the technique in a practical situation consider figure (a) below, which shows a semi dark room viewed from a doorway. Figure (b) shows the image after histogram equalisation and figure (c) is the result of interactive histogram specification. Figure (d) shows the histograms:

- from bottom to top, the original, equalised, specified and resulting histograms, respectively.

a

b

c

d
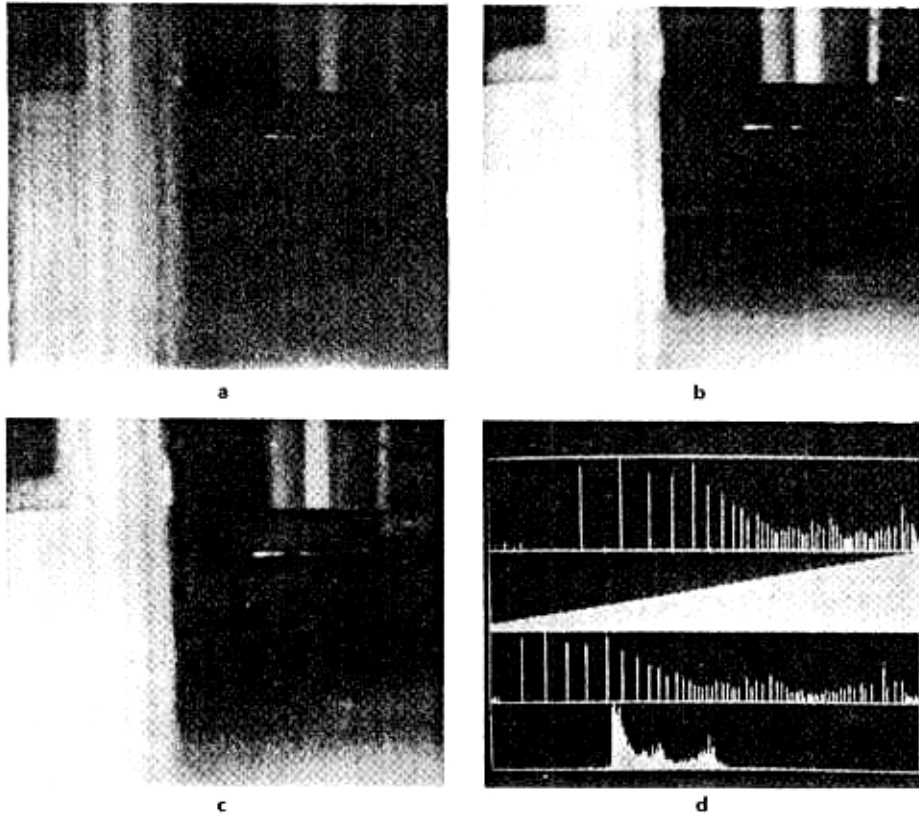
GRAY LEVEL THRESHOLDING

In the beginning we gave the histogram as:
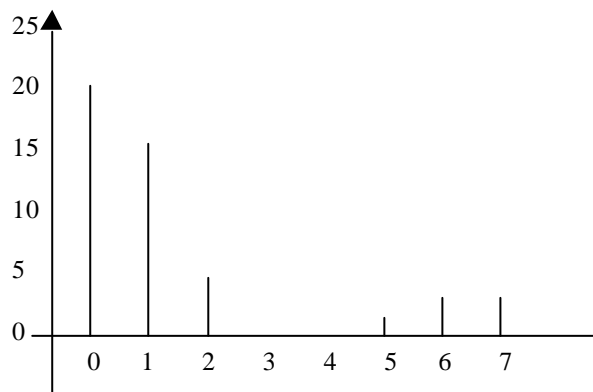
If we were to set gray levels g_k to zero when $k < 5$ our histogram consists only of the lower peak to which the corresponding image is:

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	7	5	6	6	0	0
0	0	6	5	5	7	0	0
0	0	7	7	7	6	0	0
0	0	6	6	6	5	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

The net result of this operation is that the gray levels which comprise the square in the image are enhanced relative to the background. On a graphics display of the image therefore, the square would generally appear as more distinct than in a display of the original image, due to the fact that levels constituting the background have been reduced relative to those constituting the square.

This technique of setting certain gray levels to zero relative to others is commonly called *"thresholding"*. The effectiveness of the method generally depends upon the histogram of the gray level distribution on the original image exhibiting at least two identifiable peaks, so that at least one or other of the levels contributing to the peaks can be set to zero. In such cases *the contrast* in the image is generally enhanced.

IMAGE SMOOTHING

This approach *renders* the image more acceptable to the eye by reducing the contribution made by sharp transitions, or the occurrence of spurious peaks which may arise from poor sampling or errors occurring in image transmission. The techniques involved in smoothing may be based on *spatial as well as frequency* (Fourier) domains. For our purposes we present only the spatial approach.

Neighbourhood Averaging

Neighbourhood averaging is simply a process by which a pixel gray level may be replaced by the average of the gray levels of the surrounding pixels over a specified area.

Consider a subsection of an image represented by the set of gray levels shown below:

0	1	0	2	2	1	2	0
0	0	2	7	7	7	1	0
1	1	1	2	0	1	2	0

Notice the three gray levels corresponding to the value 7 which represents a sharp transition in levels in the image subsection by comparison to the other levels.

Defining a neighbourhood or area (sometimes called a window) around a specified gray level (denoted by 0 below) as the eight surrounding pixels (denoted by X), then our averaging domain can be represented as:

X	X	X
X	0	X
X	X	X

If we position the above window so that the 0 coincides with the first gray level with value 7 and then determine the average including an gray levels occurring at position X, the value of 7 would be replaced by the value:

$$(0 + 2 + 2 + 7 + 0 + 2 + 1 + 2)/8 = 2$$

If we now shift our window one pixel to the right and continue the process the next gray level value is determined as:

$$(2 + 2 + 1 + 7 + 1 + 0 + 2 + 7)/8 = 3 \qquad \text{on digital rounding.}$$

Again with a further shift to the right our final value will be determined under the same process to be 3, so that on replacement of existing levels by those determined above our new image subsection becomes:

0	1	0	2	2	1	2	0
0	0	2	2	3	2	1	0
1	1	1	2	0	1	2	0

In other words, *the sharp transition* has been eliminated.

Mathematically the above process may be expressed as:

$$f'(x, y) = \frac{1}{N} \sum_{p,q \in S} f(p,q)$$

for x, y = 0,1, ... N-1

In the above S represents the set of co-ordinates of points in the neighbourhood of the point (x, y), excluding the point x, y itself and **N** is the total number of points in the neighbourhood.

IMAGE SHARPENING

In the previous section we showed that by adding or averaging gray levels we can achieve a smoothing effect to the image. Here we show, conversely, that by *subtracting* neighbouring gray levels we can achieve a sharpening or highlighting effect at different parts of the image.

This process of subtracting gray level values generates a further set of values commonly referred to as gradient values. Two approaches using subtraction methods are defined in relation to a pixel gray level at location *x, y* as follows:

1. form the absolute difference between the gray level at position x, y and adjacent levels at locations x + 1, y and x, y + 1 then add the result;
2. form the absolute difference between the gray level at x, y with at x + 1, y + 1 and add the result to the difference between gray levels at x, y + 1 and x + 1, y.

(1) and (2) above are represented diagramatically below:

(x, y)   (x + 1, y)   (x, Y)   (x + 1, Y)
O ←——→ O   O   O

O(x, y + 1)   O(x, y + 1)   O(x + 1, y + 1)
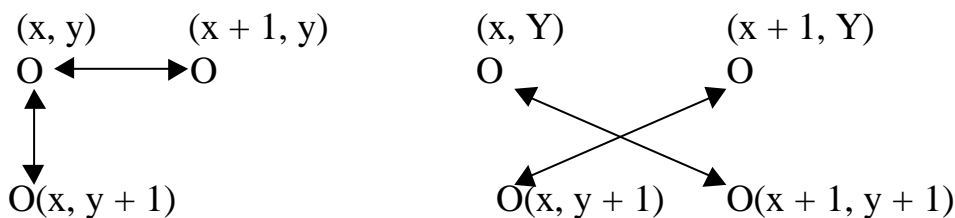
Mathematically, the operations can be expressed in terms of the gray levels g(x, y), g(x + 1, y) etc. as:

(1) G[g(x, y)] |g(x, y) - g(x + 1, y)| + |g(x, y) - g(x, y + 1)|
(2) G[g(x, y)] |g(x, y) - g(x + 1, y + 1)| + |g(x + 1, y) - g(x, y + 1)|

In the above [G(x, y)] denotes the gradient and the approximation sign has been used as the values determined above are not strictly equal to the true gradient which mathematically is:

$$G[g(x, y)] = \sqrt{[(\partial g / \partial x)^2 + (\partial g / \partial y)^2]}$$

However, for practical purposes the above approximation to the gradient is computationally faster whilst yielding acceptable results.

Having determined our gradients [G(x,y)] for all positions of x,y we may then form new images ie replace our original set of gray levels by the gradient values themselves, so forming two images for the set of g(x,y) where:

g(x, y) = G[g(x, y)]

for the two types of gradient defined above.

In the case of ***image borders*** where we have no pixels outside the actual border with which to specify the window, we simply repeat the gradient values determined at one pixel within the border.

From what has been described above, it should be fairly obvious that for image regions, other than background, consisting of similar gray level values, the difference determined within the region will give gray levels of low magnitude and probably, therefore, more consistent with the levels forming the background.

The effect of the operation at ***an edge*** say, or where we transfer from background to a region of higher gray level values, is to leave the edge values ***relatively unchanged***. The method is therefore attractive as a way of highlighting edge regions in an image.
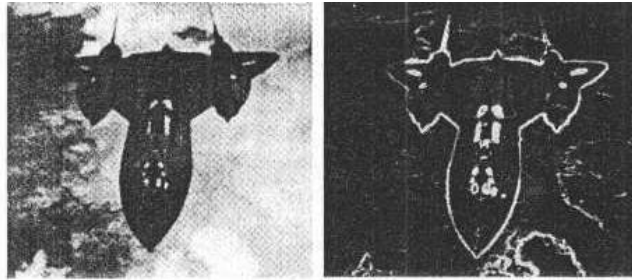
Image Segmentation and Analysis

Image processing can be broadly classified into two main areas:

ENHANCEMENT AND ANALYSIS

We concerned with image processing presenting a number of techniques directly related to the processes of enhancement.

Now we are largely concerned with the presentation of techniques of analysis. For example, consider the image

which shows the periodic structure within the image obtained through the Fourier filtering process.

This image, as presented to a scientist working in the field of virology, may cause questions to be asked about the extent and uniformity of the structure within the image.

Similarly, a meteorologist presented with a satellite image showing various weather formations may require further information about the particular size and

distribution of specific cloud formations in relation to the area under consideration.

To assist in providing answers to such questions requires that we firstly further process the image in a way which renders it suitable for subsequent analysis. This form of processing requires that we ***initially identify and isolate*** the components of interest within the image.

The above process is generally referred to as ***segmentation***. The segmentation approach is based on the ***detection of changes in the gray level distribution*** within the image. For example, a boundary can be considered as a line of pixels whose gray levels are distinctly different from the gray levels of the pixels either side of it. Detection of boundaries within an image can be achieved therefore, by identification of groups of pixels with this characteristic.

DETECTION OF POINTS, LINES AND EDGES

As pointed out above, a region in an image may be characterised by a significant change in the gray level distribution in a specified direction. For example, consider the single line of gray levels below:

0 0 0 0 0 0 0 7 0 0 0 0 0 0 0

Given the information that our background levels are represented by gray levels having a value zero, we may conclude that at the ***eighth*** pixel from the left we have a non background level and that this level may also characterise a single point.

Basically the gray level at the eighth position represents a ***discontinuity*** in that all the other levels are of uniform value ie zero, except for the pixel at position eight. The process of ***recognising such discontinuities*** particularly when they occur in groups, may be extended to the detection of lines within an image.

Most line and point detection algorithms involve the positioning of a 'window' or matrix of values at successive pixel positions within the image and then forming products with the values in the matrix and gray levels at successive pixels.

A typical matrix is illustrated below and consists of a 3 by 3 set of values v_1 to v_9

v_i	v_2	v_3
v_4	v_5	v_6
v_7	v_8	v_9

Initially the matrix is located to fit into the top left corner of the image with v_5 positioned at one pixel to the right and one pixel below the top leftmost pixel of the image. Particular values are ***ascribed*** to the elements $v_1 \dots v_9$ (as for example, below). If we denote the gray levels in the image corresponding to the matrix values $v_1, \dots v_9$ by $g_1, \dots g_9$ we start the detection process by forming the product:

$$\boldsymbol{P = v_1\,g_1 + v_2\,g_2 + \dots + v_9\,g_9}$$

We then shift our mask one pixel to the right and repeat the calculation. The process is repeated for all the pixels in the line (with the exception of the edge pixels). We then reposition the matrix one pixel down and at the left edge of the image and repeat the procedure. Note that borders are ignored, particularly as many images are of dimension ***256 by 256*** or more and the effect of ignoring the outside edge is generally insignificant, for iinages of these dimensions.

At the end of the entire process we have a set of P values from which we are able to determine the existence of a line. The basic rule for establishing the existence of a line is whether among the set of P values subsets of P values can be found for which:

$$P \geq T \text{ (where T corresponds to some arbitrary value).}$$

Below are shown a set of matrices for detecting ***lines*** oriented horizontally, vertically, and at $+ 45^0$, $- 45^0$:

-1	-1	-1		-1	-2	-1		-1	-1	2		2	-1	-1
2	2	2		-1	2	-1		-1	2	-1		1	2	-1
-1	-1	-1		-1	2	-1		2	-1	-1		-1	-1	2

horizontal	vertical	$+45^0$	$- 45^0$
detector	detector	detector	detector

Detection of Isolated Points

The process for the detection of isolated points is very much the same as that described for lines except that the matrix values are assigned a different set of values to those assigned for the detection of lines. One favoured set of values is shown below:

$$\begin{vmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{vmatrix}$$

To illustrate the technique consider the distribution of gray levels below:

$$0\ 1\ 1\ 1\ 1\ 6\ 0\ 1\ 0\ 0\ 2\ 0$$

Positioning our matrix at the top left would give for the first evaluation of P in accordance with equation given, ie forming products and adding:

$$P = (-1)\ 0 + (-1)\ 1 + (-1)\ 1 + (-1)\ 1 + (6)\ 8 + (-1)\ 0 + (-1)\ 0 + (-1)\ 0 + (-1)\ 2 = 43$$

Shifting the matrix one pixel along yields for P:

$$P = (-1)\ 1 + (-1)\ 1 + (-1)\ 1 + (-1)\ 6 + (0)\ 8 + (-1)\ 1 + (-1)\ 0 + (-1)\ 2 + (-1)\ 0 = -12$$

In general we use P and say that a single point has been detected if $P \geq T$ where T, is some arbitrary value selected on the basis of the a-prior knowledge of the image application area.

As can be seen from the example above, the value determined for P in the image is significantly higher at the position corresponding to the highest gray level value, consistent with a level constituting a point.

The Hough Transform

This transformation ***enables the automatic detection*** of lines, ***straight or curved***, in any orientation. The problem of detection may be formulated as that of locating all the points in an image which identify with straight lines or curves.

Mathematically, the problem of locating lines given a set of points is a fairly simple one although standard approaches become computationally prohibitive for all but the simplest images.

An alternative approach, proposed by Hough (1962) has the attraction of significantly ***reducing the computational overheads*** incurred by the conventional approach.

The equation for the straight line connecting a set of pixels is given by the formula:

$$y = mx + c$$

where ***m*** corresponds to the slope of the line and c its intercept with the y axis. The approach adopted by Hough was to reformulate the above equation as:

$$c = y_i - m\ x_i$$

and to restate the problem in the following way:

- Given a specific value for ***m*** and a set of coordinate pairs $x_i,\ y_i$ determine the corresponding value for c from equation above.

We can then say that for those $x_i,\ y_i$ and a specified ***m*** which yield the same values for ***c*** that we have located the line (slope ***m***, intercept ***c***) associated with those points.

The problem with the method is that for lines either vertical or tending towards the vertical, the intercept value for ***c*** tends to infinity which clearly leads to computational problems. To overcome this the normal representation of the line is used:

$$x\ cos\ è + y\ sin\ è = p$$
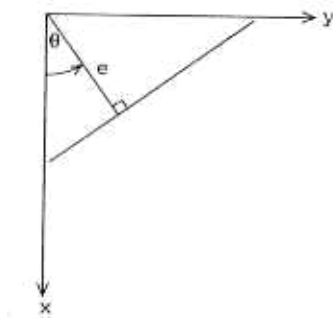
Normal representation of line

The problem may now be regarded as the determination of p for a specified value of è from the x_i, y_i values inserted into equation above.

In practice however, we are generally more concerned with the identification of points that are associated with particular lines. The coordinate pairs together with the specified values when inserted into equation above will generate a range of values for p. We may then say that those values of p which lie within a specified radius or neighbourhood are associated with the line defined by the p, è values determined by the point at the centre of the neighbourhood.

An example is given to illustrate the process of the Hough transform in relation to two points, however it should be noted before studying the example that a property of the transform is to map single points with x, y coordinates into sinusoidal curves in $p, è$ space.

For example, we have chosen a situation represented by two points A, B with respective co-ordinate values (l,l) ; (2,2) as shown in figure:
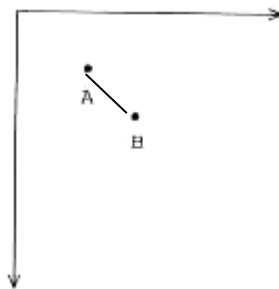
Two points to be mapped by Hough transform

Suppose we select $è_{min}$ **a**s -90^0 and $è_{max}$ as $+90^0$ and decide to increment è in steps of $+45^0$. Then for each of the points A, B we can establish the following table for the values defined by equation:

For point A:
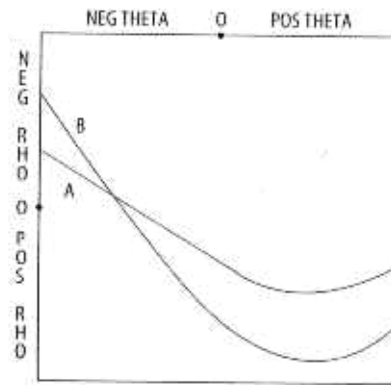
θ	$x \cos \theta$	$y \sin \theta$	ρ
$-90°$	0	-1	-1
$-45°$	$1/\sqrt{2}$	$-1\sqrt{2}$	0
$0°$	1	0	1
$+45"$	$1/\sqrt{2}$	$1/\sqrt{2}$	$\sqrt{2}$
$+90°$	0	1	1

For point B:

| θ	$x\cos\theta$	$y\sin\theta$	ρ
$-90°$	0	-2	-2
$-45°$	$2/\sqrt{2}$	$-2/\sqrt{2}$	0
$0°$	2	0	2
$+45°$	$2/\sqrt{2}$	$2/\sqrt{2}$	$2\sqrt{2}$
$+90°$	0	2	2

The results may be interpreted as follows:

We firstly observe that for the values corresponding to $p = 0$ and $\theta = 45^0$, the two curves intersect. The point of intersection identifies that the two curves (or points in x, y coordinate space) are common to the points having a line of slope -45^0 with perpendicular bisector to the line having a slope of zero.

This result is clearly confirmed by the line joining the two points A and B.

The method may be extended to the detection of points associated with analytically defined curves.

IMAGE TEXTURE

The texture of an image can be defined in terms of its smoothness or its coarseness sometimes called graininess. One area of image processing in which the quantification of texture, as defined above, plays a crucial role is that of industrial vision.
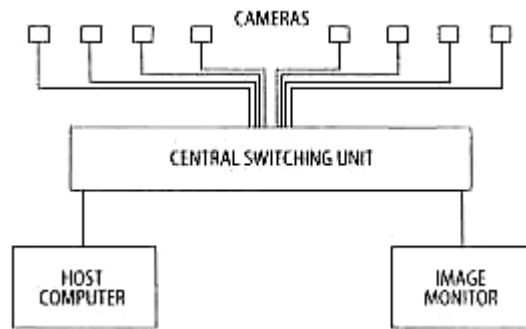
These systems may be used to assess the quality of products by measuring the **_texture of the surface_** of the product. In general a poor surface, such as one showing a high degree of graininess, implies a poor product. The basis of industrial vision systems is on the use of intelligent cameras and appropriate software for image analysis.

The attraction in using an industrial vision system of the above type is that they are more sensitive than manual approaches to the identification of surface defects. Furthermore, manual inspection systems are subject to varying standards and involve fairly high costs. These factors together with the degree of subjectivity involved in using a manual approach makes the vision systems an attractive alternative.

We are to explain two methods for the identification and assessment of texture for a specified region. The methods are based on:

- the statistical properties of an image;
- the spectral or Fourier characteristics of the image.

The use of statistical techniques enables the characterisation of the texture in terms of smoothness or coarseness whereas the spectral approach is based on the properties of the Fourier spectrum and is used to detect regular or periodic features which may be present in the image. Examples showing images exhibit different textural features:
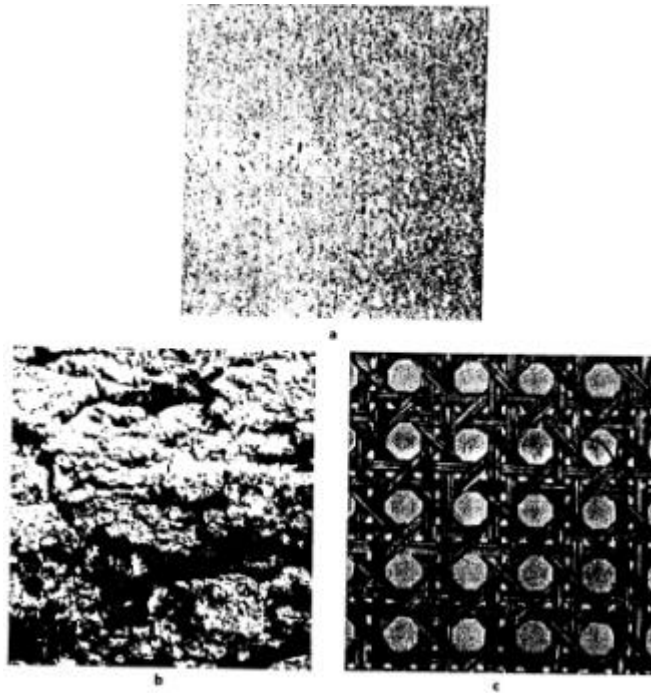
Statistical Methods for Texture Description

For an image with a corresponding histogram distribution of gray levels $p(z_i)$ where z denotes the ith gray level and $p(z_i)$ its frequency, the nth moment of z about the mean m can be written as:

$$m_n(z) = \sum_{i=1}^{L} (z_i - m)^n \, p(z_i)$$

where L is a number of distinct levels.

Note that $\mu_0 = 1$ and $\mu_1 = 0$.

The second moment, sometimes called the variance $\sigma^2(z)$, determines gray-level *contrast* and may be used to establish descriptions of relative smoothness. In particular the value for R in equation below is zero for areas of constant intensity and approaches unity for large values of cy,(Z)

$$R = 1 - (1/(1 + \sigma^2(z)))$$

Additional information such as the *skewness* of the histogram can be obtained from the *third* moment.

The *fourth moment* provides details on the relative flatness.

The *fifth and higher* moments do not provide much information about histogram shape but do provide further quantitative discrimination of texture content.

Fourier or Spectral Methods for Texture Description

Determination of the Fourier spectrum provides a useful way of identifying any periodic or regular features within an image.

Three features of the spectrum that have been found useful for texture description are:

  the significant peaks in the spectrum give the principal direction of the texture patterns;
  the location of the peaks in frequency space provide the spatial period of the patterns;
  filtering of periodic components leaves nonperiodic image elements which may then be described by statistical techniques.

In fact this latter step corresponds to the converse filtering operation (used to generate the biological image in previous lecture) in which the nonperiodic components of the spectrum were filtered out prior to involving the inverse transform operation.
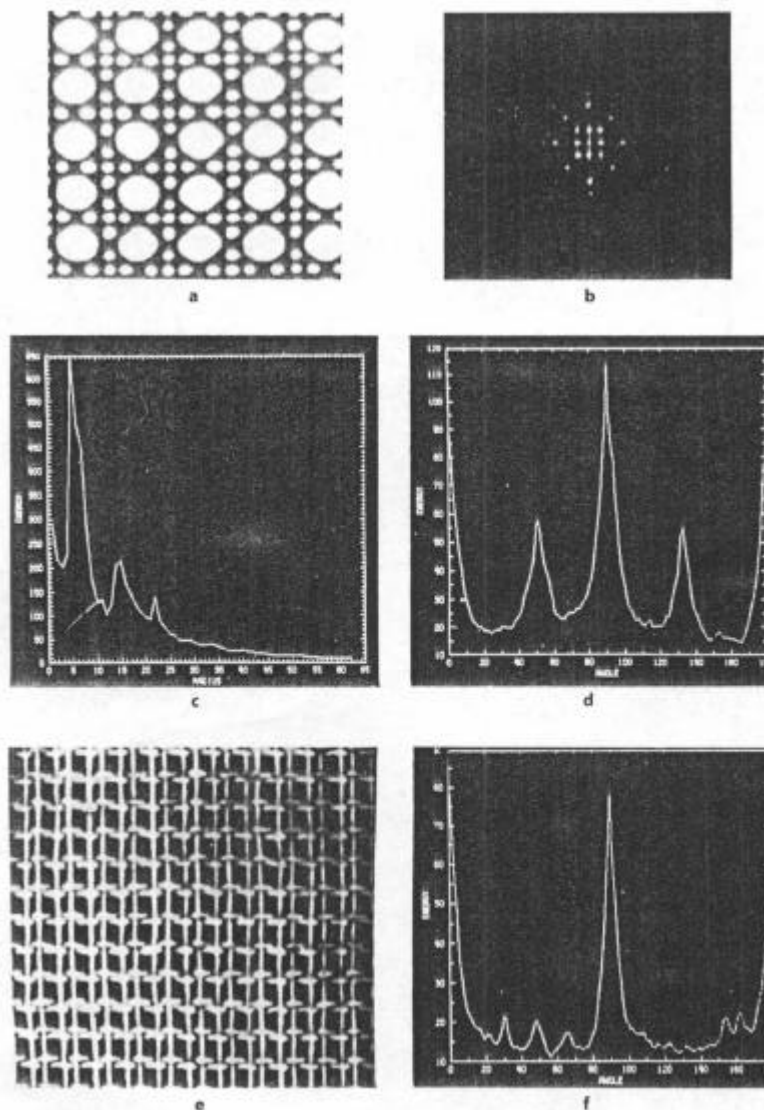
(a)periodic image, (b)spectrum, (c)plot of *S(r)*, (d)plot of *S(è)*, (e)another texture, (f) plot of *S(è)*

The approach to abstracting and analyzing the features in the spectrum is to express the spectrum in polar coordinates to yield a function *S(r, è)*, where *S* is the spectrum function and *r* and *è* are the variables in the coordinate system.

For each direction *è* and each frequency *r*, both *S(r, è)* and $S_è(r)$ are considered to be one dimensional functions. $S_è(r)$ for a fixed value of *è* yields spectral characteristics along a radial direction from the origin, whereas $S_r(è)$ for a fixed value of *r* yields the behaviour along a circle centred on the origin

A more global description is obtained using the functions:

$$S(r) = \sum_{\boldsymbol{q}=0}^{\boldsymbol{p}} S_{\boldsymbol{q}}(r)$$

$$S(\boldsymbol{q}) = \sum_{r=1}^{R} S_r(\boldsymbol{q})$$