

11 pratybos. Ortogonaliosios transformacijos \mathbb{R}^3 ir jų matricos

Uždavinys. Pasukite vektorių \mathbf{x} kampų α vektoriaus \mathbf{u} kryptimi

1 $\mathbf{u} = (1, 1, 0)$, $\alpha = 20^\circ$, $\mathbf{x} = (0, 0, 20)$

Ats.: (4.8; -4.8; 18.8)

2 $\mathbf{u} = (-1, 1, 0)$, $\alpha = 20^\circ$, $\mathbf{x} = (0, 0, -20)$

Ats.: (-4.8; -4.8; -18.8)

3. $\mathbf{u} = (-1, -1, 0)$, $\alpha = 40^\circ$, $\mathbf{x} = (0, 0, 30)$

Ats.: (-13.6; 13.6; 23.0)

4. $\mathbf{u} = (1, -1, 0)$, $\alpha = 40^\circ$, $\mathbf{x} = (0, 0, -30)$

Ats.: (13.6; 13.6; -23.0)

5. $\mathbf{u} = (1, 0, 1)$, $\alpha = 50^\circ$, $\mathbf{x} = (0, 0, 40)$

Ats.: (7.1; -21.7; 32.9)

6. $\mathbf{u} = (-1, 0, 1)$, $\alpha = 50^\circ$, $\mathbf{x} = (0, 0, -40)$

Ats.: (7.1; -21.7; -32.9)

7. $\mathbf{u} = (-1, 0, -1)$, $\alpha = 70^\circ$, $\mathbf{x} = (0, 0, 50)$

Ats.: (16.5; 33.2; 33.6)

8. $\mathbf{u} = (1, 0, -1)$, $\alpha = 70^\circ$, $\mathbf{x} = (0, 0, -50)$

Ats.: (16.5; 33.2; -33.6)

9. $\mathbf{u} = (0, -1, -1)$, $\alpha = 80^\circ$, $\mathbf{x} = (0, 0, 60)$

Ats.: (-41.8; 24.8; 35.2)

Pavyzdys

Pasukite vektorių \mathbf{x} kampą α vektoriaus \mathbf{u} kryptimi

Duota:

$$\mathbf{u} = (0, -1, 1), \alpha = 80^\circ, \mathbf{x} = (0, 0, -60)$$

Sprendimas.

$$\sin 80^\circ = 0.98, \cos 80^\circ = 0.17$$

$$\mathbf{u} = (0, -1, 1), \mathbf{v} = (1, 0, 0), \mathbf{w} = \mathbf{u} \times \mathbf{v}$$

$$\mathbf{u} \times \mathbf{v} = \det \begin{pmatrix} i & j & k \\ 0 & -1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = j + k = (0, 1, 1)$$

$$\begin{pmatrix} 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 80^\circ & -\sin 80^\circ \\ 0 & \sin 80^\circ & \cos 80^\circ \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}^T$$
$$= \begin{pmatrix} 0.17365 & -0.69636 & -0.69636 \\ 0.69636 & 0.58682 & -0.41318 \\ 0.69636 & -0.41318 & 0.58682 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -60 \end{pmatrix} = \begin{pmatrix} 41.782 \\ 24.791 \\ -35.209 \end{pmatrix}$$

$$\text{Ats.: } \begin{pmatrix} 41.8 \\ 24.8 \\ -35.2 \end{pmatrix}^T = (41.8; 24.8; -35.2)$$