Algebros pratybos.

Rimantas Grigutis

9 pratybos. Tiesinės transformacijos $\mathbf{R}^n$

1. Raskite tiesinės transformacijos $\mathcal{A}$ matricą, jei
$\mathcal{A}(\mathbf{u}_1) = \mathbf{v}_1, \mathcal{A}(\mathbf{u}_2) = \mathbf{v}_2, \mathcal{A}(\mathbf{u}_3) = \mathbf{v}_3$

1.1
$$\begin{aligned}
\mathbf{u}_1 &= (1; 2; 1) & \mathbf{v}_1 &= (-3; 3; 6) \\
\mathbf{u}_2 &= (2; 1; -1) & \mathbf{v}_2 &= (-8; 4; 4) \\
\mathbf{u}_3 &= (-1; 1; 2) & \mathbf{v}_3 &= (5; 7; 2)
\end{aligned}$$

1.2
$$\begin{aligned}
\mathbf{u}_1 &= (1; -2; 3) & \mathbf{v}_1 &= (-3; 0; 5) \\
\mathbf{u}_2 &= (-1; 2; -2) & \mathbf{v}_2 &= (-3; 9; -7) \\
\mathbf{u}_3 &= (-2; 5; -6) & \mathbf{v}_3 &= (-3; 6; -6)
\end{aligned}$$

2. Raskite tiesinės transformacijos $\mathcal{A}$ matricą bazėje $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, jei matrica bazėje $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ yra A

2.1 $A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ -2 & 4 & 3 \end{pmatrix}$
$\qquad \begin{aligned} \mathbf{v}_1 &= -2\mathbf{u}_1 + 3\mathbf{u}_2 - 3\mathbf{u}_3 \\ \mathbf{v}_2 &= -\mathbf{u}_1 + 4\mathbf{u}_2 - 2\mathbf{u}_3 \\ \mathbf{v}_3 &= -3\mathbf{u}_1 + 6\mathbf{u}_2 - 5\mathbf{u}_3 \end{aligned}$

2.2 $A = \begin{pmatrix} 2 & 3 & -4 \\ 1 & 2 & -1 \\ -3 & -2 & 3 \end{pmatrix}$
$\qquad \begin{aligned} \mathbf{u}_1 &= (2; 4; -1) & \mathbf{v}_1 &= (21; 15; 13) \\ \mathbf{u}_2 &= (-7; 1; 3) & \mathbf{v}_2 &= (-13; -5; -5) \\ \mathbf{u}_3 &= (1; -1; 5) & \mathbf{v}_3 &= (26; 22; 17) \end{aligned}$

3. Raskite $\ker\mathcal{A}$ ir $\operatorname{im}\mathcal{A}$ bazes, jei tiesinės transformacijos $\mathcal{A}$ matrica yra A.

3.1 $A = \begin{pmatrix} 1 & 2 & 1 & 3 \\ 2 & 0 & -1 & 4 \\ 0 & -4 & -3 & -2 \\ 3 & -2 & -3 & 5 \end{pmatrix}$
$\qquad$
3.2 $A = \begin{pmatrix} 1 & -1 & -1 & -1 \\ 3 & -2 & 0 & 3 \\ 0 & 1 & 1 & -3 \\ 1 & 1 & -2 & 2 \end{pmatrix}$