Algebra pratybos.

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8 pratybos. Euklido erdvės. Metrinių erdvių geometrija.

8.1 Apibrėžimas. *Tegu U - Euklido erdvės E poerdvis. Su kiekvienu $\mathbf{v} \in E$ egzistuoja $\mathbf{u} \in L$ ir toks $\mathbf{w} \perp U$, kad $\mathbf{v} = \mathbf{u} + \mathbf{w}$. Čia $\boldsymbol{u}$ vadinamas vektoriaus $\boldsymbol{v}$ ortogonaliąja projekcija,* $\mathbf{u} =\text{proj}_U\mathbf{v}$ *, o $\boldsymbol{w}$ - vektoriaus $\boldsymbol{v}$ statmeniu į U,* $\mathbf{w}=\text{ort}_U\mathbf{v}$.

8.2 Apibrėžimas. *Atstumas tarp vektoriaus $\mathbf{v}$ ir poerdvio U - tai* $\text{dist}_U\mathbf{v} = \|\text{ort}_U\mathbf{v}\|$

8.3 Apibrėžimas. *Kampas tarp vektoriaus $\mathbf{v}$ ir poerdvio U - tai kampas tarp $\mathbf{v}$ ir* $\text{proj}_U\mathbf{v}$: $\cos\left(\widehat{\mathbf{v}\ \ proj_U\mathbf{v}}\right) = \dfrac{(\mathbf{v},\text{proj}_U\mathbf{v})}{\|\mathbf{v}\|\,\|\text{proj}_U\mathbf{v}\|}$

1. Raskite tiesinio apvalkalo $[\mathbf{u}_1, ..., \mathbf{u}_m]$ bazę ir ortogonalizuokite ją.

1.1 $\mathbf{u}_1 = (1, 2, 1)\,, \mathbf{u}_2 = (-3, -4, -1)\,, \mathbf{u}_3 = (-4, -7, 0)\,.$
1.2 $\mathbf{u}_1 = (2, 3, -4)\,, \mathbf{u}_2 = (-3, -1, 5)\,, \mathbf{u}_3 = (8, -13, 16)$
1.3 $\mathbf{u}_1 = (1, 0, 0, 0)\,, \mathbf{u}_2 = (0, 2, 0, 0)\,, \mathbf{u}_3 = (0, 0, 3, 0)\,, \mathbf{u}_4 = (0.0.0.4)\,.$
1.4 $\mathbf{u}_1 = (1, 0, 1, 0)\,, \mathbf{u}_2 = (0, 1, 2, 0)\,, \mathbf{u}_3 = (0, 0, 1, 0)\,, \mathbf{u}_4 = (0, 0, 3, 1)\,.$
1.5 $\mathbf{u}_1 = (1, 1, 1, 1)\,, \mathbf{u}_2 = (0, 1, 1, 1)\,, \mathbf{u}_3 = (0, 0, 1, 1)\,, \mathbf{u}_4 = (0, 0, 0, 1)\,.$
1.6 $\mathbf{u}_1 = (1, 1, 1, 1)\,, \mathbf{u}_2 = (1, 1, -1, -1)\,, \mathbf{u}_3 = (1, -1, 1, -1)\,, \mathbf{u}_4 = (1, -1, -1, 1)$

2. Vektorių sistemą papildykite iki R^n ortonormuotosios bazės.

2.1 $\mathbf{u}_1 = \frac{1}{3}(1, -2, 2)\,, \mathbf{u}_2 = \frac{1}{3}(-2, 1, 2)\,.$
2.2 $\mathbf{u}_1 = (2, 3, -4, -6)\,, \mathbf{u}_2 = (1, 8, -2, -16)\,, \mathbf{u}_3 = (1, -5, -2, 10)$

3. Raskite tiesinio apvalkalo U ortogonaliojo papildinio bazę.

3.1 $U = [(4, 1, 2, -3)\,, (2, -2, 3, 5)]\,.$
3.2 $U = [(3, 4, -4, -1)\,, (0, 1, -1, 2)]\,.$

4. Raskite $\text{proj}_U\mathbf{v}$, $\text{ort}_U\mathbf{v}$, $\text{dist}_U\mathbf{v}$, $\widehat{\mathbf{v}\ \ \text{proj}_U}\mathbf{v}$.

4.1 $\mathbf{v} = (2, -3, 3, -3)$, $U = [(1, -1, 2, 3)\,, (-1, 3, 1, 5)]$
4.2 $\mathbf{v} = \left(3, 1, \sqrt{2}, -2\right)$; $U = [(2, -1, 2, 1)\,, (-1, 2, -2, 1)\,, (-1, 1, -1, 0)]$

5. Raskite $\text{proj}_U\mathbf{v}$, $\text{ort}_U\mathbf{v}$
5.1 $\mathbf{v} = (14, -3, -6, -7)\,; U = [(-3, 0, 7, 6)\,; (1, 4, 3, 2)\,; (2, 2, -2, -2)]\,.$
5.2 $\mathbf{v} = (2, -5, 3, 4)\,; U = [(1, 3, 3, 5)\,, (1, 3, -5, -3)\,, (1, -5, -3, -3)]\,.$

5.3 $\mathbf{v} = (-3, 0, -5, 9)\,, U = \left\{ (a_1, a_2, a_3, a_4) : \left\{ \begin{array}{c} 3a_1 + 2a_2 + a_3 - 2a_4 = 0 \\ 5a_1 + 4a_2 + 3a_3 + 2a_4 = 0 \\ a_1 + 2a_2 + 3a_3 + 10a_4 = 0 \end{array} \right. \right\}.$

5.4 $\mathbf{v} = (7, -4, -1, 2)\,, U = \left\{ (a_1, a_2, a_3, a_4) : \left\{ \begin{array}{c} 2a_1 + a_2 + a_3 + 3a_4 = 0 \\ 3a_1 + 2a_2 + 2a_3 + a_4 = 0 \\ a_1 + 2a_2 + 2a_3 - 4a_4 = 0 \end{array} \right. \right\}.$

6. Raskite tiesinę lygčių sistemą, kurios sprendinių poerdvis yra poerdvio U ortogonalusis papildinys.

6.1 $U = \left\{ (x_1, x_2, x_3, x_4) : \left\{ \begin{array}{c} 2x_1 + x_2 + 3x_3 - x_4 = 0 \\ 3x_1 + 2x_2 - 2x_4 = 0 \\ 3x_1 + x_2 + 4x_3 - x_4 = 0 \end{array} \right. \right\}.$

6.2 $U = \left\{ (x_1, x_2, x_3, x_4) : \left\{ \begin{array}{c} 2x_1 - 3x_2 + 4x_3 - 4x_4 = 0 \\ 3x_1 - x_2 + 11x_3 - 13x_4 = 0 \\ 4x_1 + x_2 + 18x_3 - 23x_4 = 0 \end{array} \right. \right\}.$

7. Apskaičiuokite gretasienio tūrį, kurio kraštinėmis yra vektoriai:

7.1 $\begin{array}{l} \mathbf{u}_1 = (1, -1, 1, -1) \\ \mathbf{u}_2 = (1, 1, 1, 1) \\ \mathbf{u}_3 = (1, 0, -1, 0) \\ \mathbf{u}_4 = (0, 1, 0, -1) \end{array}$

7.2 $\begin{array}{l} \mathbf{u}_1 = (1, 1, 1, 1) \\ \mathbf{u}_2 = (1, -1, -1, 1) \\ \mathbf{u}_3 = (2, 1, 1, 3) \\ \mathbf{u}_4 = (0, 1, -1, 0) \end{array}$

7.3 $\begin{array}{l} \mathbf{u}_1 = (1, 1, 1, 2, 1) \\ \mathbf{u}_2 = (1, 0, 0, 1, -2) \\ \mathbf{u}_3 = (2, 1, -1, 0, 2) \\ \mathbf{u}_4 = (0, 7, 3, -4, -2) \\ \mathbf{u}_5 = (39, -37, 51, -29, 5) \end{array}$

7.4 $\begin{array}{l} \mathbf{u}_1 = (1, 0, 0, 2, 5) \\ \mathbf{u}_2 = (0, 1, 0, 3, 4) \\ \mathbf{u}_3 = (0, 0, 1, 4, 7) \\ \mathbf{u}_4 = (2, -3, 4, 11, 12) \\ \mathbf{u}_5 = (0, 0, 0, 0, 1) \end{array}$.