Mathematical Competition for Students (MIFMO)
of the Department of Mathematics and Informatics
of Vilnius University

2021-02-13

(organized by Paulius Drungilas and Artūras Dubickas)

Problem 1. Does there exist an uncountable set of real numbers S such that

$$|s_1 + \cdots + s_n| < 10$$

for every $n \in \mathbb{N}$ and every finite subset $\{s_1, \ldots, s_n\}$ of S?

Problem 2. Find all functions $f : \mathbb{Z} \mapsto \mathbb{Z}$ satisfying

$$f(x + y) = f(x) + f(y) + 2xy$$

for $x, y \in \mathbb{Z}$ and $f(100) = 1000$.

Problem 3. Alex and Sveta play a game in which they take turns filling entries of an initially empty table with 2020 rows and 2021 columns. Alex plays first. At each turn, a player chooses a number and places it in a vacant entry. Alex can only choose numbers of the form $a\sqrt{2}$, with $a \in \mathbb{Z}$, while Sveta can only choose rational numbers. The game ends when all the entries are filled. Alex wins if the rank of the resulting matrix is at least 1011, Sveta wins if the rank is at most 1010. Which player has a winning strategy?

Problem 4. Find all continuous functions $f : [1, 8] \mapsto \mathbb{R}$ satisfying

$$10 \int_1^8 f(x)dx - 15 \int_1^2 f^2(x^3)dx - 30 \int_1^2 f(x^3)dx = 38.$$

Each problem is worth 10 points.