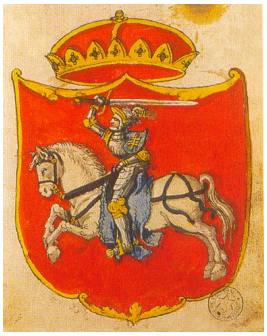
# $10^{\text{th}}$ Mathematical Contest of Friendship in Honor and Memory of Grand Duchy of Lithuania

**30 September 2018**



1. Let $x, y, z, t$ be real numbers such that

$$(x^2 + y^2 - 1)(z^2 + t^2 - 1) > (xz + yt - 1)^2.$$

Prove that

$$x^2 + y^2 > 1.$$

2. Ten distinct numbers are chosen at random from the set $\{1, 2, 3, \ldots, 37\}$. Show that one can select four distinct numbers out of those ten so that the sum of two of them is equal to the sum of the other two.

3. The altitudes $AD$ ir $BE$ of an acute triangle $ABC$ intersect at point $H$. Let $F$ be the intersection of the line $AB$ and the line that is parallel to the side $BC$ and goes through the circumcenter of $ABC$. Let $M$ be the midpoint of the segment $AH$. Prove that $\angle CMF = 90^\circ$.

4. Find all positive integers $n$ for which there exists a positive integer $k$ such that for every positive divisor $d$ of $n$, the number $d - k$ is also a (not necessarily positive) divisor of $n$.