

**Mathematical Competition for Students (MIFMO)
of the Department of Mathematics and Informatics
of Vilnius University**

2017-02-11

(organized by Paulius Drungilas, Artūras Dubickas and Jonas Jankauskas)

Problem 1. Let R be the region consisting of all the points (x, y) in the Cartesian plane \mathbb{R}^2 satisfying two inequalities $|x| - y^2 \leq 1$ and $|y| \leq 1$. Find the area of R .

Problem 2. Find all prime numbers in the infinite sequence of integers

101, 10101, 1010101, 101010101, \dots

Problem 3. Let $n \geq 2$ be a fixed positive integer. Find all positive integers k with the property that the k th derivative $f^{(k)}(x)$ of an arbitrary infinitely many times differentiable function $f : \mathbb{R} \rightarrow \mathbb{R}$ with at least n distinct zeros in the interval $[0, 1]$ (namely, $f(a_1) = f(a_2) = \dots = f(a_n) = 0$ for some $a_1 < a_2 < \dots < a_n \in [0, 1]$) has at least one zero in the interval $[0, 1]$.

Problem 4. Find all pairs of positive integers (m, n) satisfying

$$m^{n^m} = n^{m^n}.$$

Each problem is worth 10 points.