

**Mathematical Competition for Students (MIFMO)
of the Department of Mathematics and Informatics
of Vilnius University**

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(organized by Paulius Drungilas, Artūras Dubickas and Jonas Jankauskas)

Problem 1. Let A be the set of all real numbers that can be written as $n - \sqrt[3]{m}$ with some $m, n \in \mathbb{Z}$.

- a) Is $\sqrt{2}$ an element of the set A ?
- b) Is $\sqrt{2}$ a limit point of the set A ?

Problem 2. Find the value of the integral

$$\int_{\pi/6}^{\pi/3} \frac{\cos x dx}{\cos x + 2 \sin x}.$$

Problem 3. Find all functions $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ satisfying

$$f(f(f(x))) + 5f(f(x)) + 2f(x) = 8x$$

for each $x \in \mathbb{R}^+$. (Here, \mathbb{R}^+ is the set of all nonnegative real numbers.)

Problem 4. Let n be a positive integer.

- a) Prove that there exists a positive integer m divisible by n whose decimal expansion contains only digits from the set $\{0, 1, 8, 9\}$.
- b) Prove that such an integer m as in part a) can be chosen in the interval $[n, n^4]$.

Each problem is worth 10 points.