VILNIUS UNIVERSITY
UNDERGRADUATE MATHEMATICS COMPETITION

2007

1. Find all non-empty finite sets S of positive integers such that if m, $n \in S$, then $(m+n)/GCD(m,n) \in S$.

2. Suppose that a 6×6 square grid of unit squares (chessboard) is tiled by 1×2 rectangles (dominoes). Prove that it can be decomposed into two rectangles, tiled by disjoint subsets of the dominoes. Is the same thing true for an 8×8 array?

3. A is a subset of a finite group G, and A contains more than one half of the elements of G. Prove that each element of G is the product of two elements of A.

4. Prove that the integral
$$\int_0^\infty \frac{\sin^2 x}{\pi^2 - x^2}\, dx$$
exists and evaluate it.