4^{th} Mathematical Contest of Friendship in Honor and Memory of Grand Duchy of Lithuania

30th September 2012

Welcomed by Professor Nazar Agakhanov, the Chairman of the Advisory Board of the International Mathematical Olympiad

1. Find all functions $g : \mathbb{R} \to \mathbb{R}$, for which there exists a strictly increasing function $f : \mathbb{R} \to \mathbb{R}$ such that
$$f(x + y) = f(x)g(y) + f(y).$$

2. The base AB of a trapezium $ABCD$ is longer than the base CD, and $\angle ADC$ is a right angle. The diagonals AC and BD are perpendicular. Let E be the foot of the altitude from D to the line BC. Prove that
$$\frac{AE}{BE} = \frac{AC \cdot CD}{AC^2 - CD^2}.$$

3. How many ways are there to line up 19 girls (all of different heights) in a row so that no girl has a shorter girl both in front of and behind her?

4. Let m be a positive integer. Find all bounded sequences of integers $a_1, a_2, a_3, \ldots$ for which $a_n + a_{n+1} + a_{n+m} = 0$ for all $n \in \mathbb{N}$.