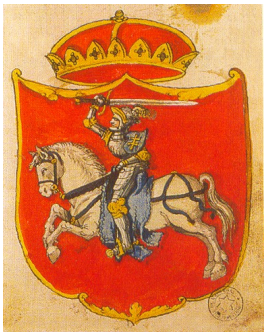
8^{th} Mathematical Contest of Friendship in Honor and Memory of Grand Duchy of Lithuania

25 September 2016



1. Let a, b and c be positive real numbers such that $a+b+c=1$. Prove that
$$\frac{a}{a+b^2}+\frac{b}{b+c^2}+\frac{c}{c+a^2}\leqslant\frac{1}{4}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right).$$

2. During a school year 44 competitions were held. Exactly 7 students won in each of the competitions. For any two competitions, there exists exactly 1 student who won in both competitions. Is it true that there exists a student who won all of the competitions?

3. Let ABC be an isosceles triangle with $AB=AC$. Let D, E and F be points on line segments BC, CA and AB, respectively, such that $BF=BE$ and such that ED is the angle bisector of $\angle BEC$.

   Prove that $BD=EF$ if and only if $AF=EC$.

4. Determine all positive integers n such that 7^n-1 is divisible by 6^n-1.