9

Springer Series on Chemical Sensors and Biosensors

Methods and Applications

Series Editor: G. Urban

Springer Series on Chemical Sensors and Biosensors

Series Editor: G. Urban

Recently Published and Forthcoming Volumes

Optical Guided-wave Chemical and Biosensors II
Volume Editors: Mohammed Z., Lakhtakia A.
Vol. 8, 2009

Optical Guided-wave Chemical and Biosensors I
Volume Editors: Mohammed Z., Lakhtakia A.
Vol. 7, 2009

Hydrogel Sensors and Actuators
Volume Editors: Gerlach G., Arndt K. -F.
Vol. 6, 2009

Piezoelectric Sensors
Volume Editors: Steinem C., Janshoff A.
Vol. 5, 2006

Surface Plasmon Resonance Based Sensors
Volume Editor: Homola J.
Vol. 4, 2006

Frontiers in Chemical Sensors
Novel Principles and Techniques
Volume Editors: Orellana G., Moreno-Bondi M. C.
Vol. 3, 2005

Ultrathin Electrochemical Chemo- and Biosensors
Technology and Performance
Volume Editor: Mirsky V. M.
Vol. 2, 2004

Optical Sensors
Industrial, Environmental and Diagnostic Applications
Volume Editors:
Narayanaswamy R., Wolfbeis O. S.
Vol. 1, 2003

Mathematical Modeling of Biosensors

An Introduction for Chemists and Mathematicians

Volume Authors: Romas Baronas · Feliksas Ivanauskas
Juozas Kulys

Springer

Chemical sensors and biosensors are becoming more and more indispensable tools in life science, medicine, chemistry and biotechnology. The series covers exciting sensor-related aspects of chemistry, biochemistry, thin film and interface techniques, physics, including opto-electronics, measurement sciences and signal processing. The single volumes of the series focus on selected topics and will be edited by selected volume editors. The Springer Series on Chemical Sensors and Biosensors aims to publish state-of-the-art articles that can serve as invaluable tools for both practitioners and researchers active in this highly interdisciplinary field. The carefully edited collection of papers in each volume will give continuous inspiration for new research and will point to existing new trends and brand new applications.

ISSN 1612-7617
ISBN 978-90-481-3242-3 e-ISBN 978-90-481-3243-0
DOI: 10.1007/978-90-481-3243-0

Springer Dordrecht Heidelberg London New York

Library of Congress Control Number: 2009931246

Cover design: SPi Publisher Services

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Series Editor

Prof. Dr. Gerald Urban

IMTEK - Laboratory for Sensors
Institute for Microsystems Engineering
Albert-Ludwigs-University
Georges-Köhler-Allee 103
79110 Freiburg
Germany
urban@imtek.de

Volume Authors

Romas Baronas
Feliksas Ivanauskas

Vilnius University
Dept. Mathematics & Informatics
Naugarduko 24
LT-03225 Vilnius
Lithuania
romas.baronas@mif.vu.lt
feliksas.ivanauskas@mif.vu.lt

Juozas Kulys

Vilnius Gediminas Technical University
Fac. Fundamental Sciences
Dept. Chemistry & Bioengineering
Sauletekio Ave. 11
LT-10223 Vilnius
Lithuania
JKulys@bchi.lt

Preface

Biosensors are analytical devices in which specific recognition of the chemical substances is performed by biological material. The biological material that serves as recognition element is used in combination with a transducer. The transducer transforms concentration of substrate or product to electrical signal that is amplified and further processed. The biosensors may utilize enzymes, antibodies, nucleic acids, organelles, plant and animal tissue, whole organism or organs. Biosensors containing biological catalysts (enzymes) are called catalytical biosensors. These type of biosensors are the most abundant, and they found the largest application in medicine, ecology, and environmental monitoring.

The action of catalytical biosensors is associated with substrate diffusion into biocatalytical membrane and it conversion to a product. The modeling of biosensors involves solving the diffusion equations for substrate and product with a term containing a rate of biocatalytical transformation of substrate. The complications of modeling arise due to solving of partially differential equations with non-linear biocatalytical term and with complex boundary and initial conditions.

The book starts with the modeling biosensors by analytical solution of partial differential equations. Historically this method was used to describe fundamental features of biosensors action though it is limited by substrate concentration, and is applicable for simple biocatalytical processes. Using this method the action of biosensors was analyzed at critical concentrations of substrate and enzyme activity. The substrates conversion in single and multienzyme membranes was studied. The different schemes of substrates conversion which found practical application for biosensors construction were analyzed. The biosensors dynamics was considered at the simplest scheme of biocatalyzer action.

The other part of the book covers digital modeling of biosensors. The biosensors based on amperometric as well as potentiometric transducers are considered. The action of biosensors containing single and multienzymes were modeled using the finite difference technique at nonstationary and steady state. Special emphasis was placed to model biosensors utilizing a complex biocatalytical conversion and biosensors with multipart transducers geometry and biocatalytical membranes structure.

The final part of the book is dedicated to the basic concepts of the theory of the difference schemes for the digital solving of linear diffusion equations which are basis for biosensors modeling.

The book can be recommended for the master and doctoral studies as well as for special studies of biosensors modeling. The Part 3 can also be used for independent study of digital solution of differential equations.

The book was prepared for the period of students teaching by R. Baronas and F. Ivanauskas at Vilnius University and by J. Kulys at Vilnius Gediminas Technical University. The authors acknowledge particular universities for the support of the manuscript preparation. The contribution of the coauthors of the cited publications is highly appreciated.

Vilnius,
February 2009

Romas Baronas
Feliksas Ivanauskas
Juozas Kulys

Acknowledgements

The authors acknowledge Vilnius University (Romas Baronas and Feliksas Ivanauskas) and Vilnius Gediminas Technical University (Juozas Kulys) for the support of monograph preparation.

Contents

Part II Numerical Modeling of Biosensors

Part III Numerical Methods for Reaction–Diffusion Equations

Introduction

The action of biocatalytical biosensors can be modeled with partial differential equations (PDE) of substrates and products diffusion and conversion in biocatalytical membranes. This book deals with biosensors modeling using analytical and digital solution of the PDE. The intrinsic logics of the book is to evaluate critical parameters and conditions that determinate the biosensors response. Since the analytical solutions of PDE describing biosensors action is possible at limited conditions the modeling of complex biosensor action are performed using digital solution of PDE.

The first part of the book is dedicated to the modeling biosensors by analytical solution of partial differential equations. First chapter of Part I contains tutorial introduction of kinetics of biocatalytical reactions, transducer function of biosensors and a general scheme of biosensor action. In second chapter of Part I the modeling biosensors at steady state and internal diffusion limitation is considered with special contribution to varies schemes of enzymes action. Third chapter of Part I concerns the modeling of biosensors at steady state and external diffusion limitations. The action of biosensor containing single enzyme, biosensors with multienzyme and biosensor utilizing non Michaelis–Menten enzyme kinetics was analyzed. Fourth chapter of Part I contains results of modeling biosensors utilizing microbial cells acting as specific biocatalytical or unspecific biochemical oxygen demand microreactor. The main task of fifth chapter of Part I is to analyze limited cases of biosensors modeling at nonstationary state at some critical concentrations of substrate when analytical solution of PDE was performed. The non stationary response of amperometric as well as potentiometric biosensor was analyzed.

At the end of the first part advantages and disadvantages of analytical modeling of biosensors are shown. The largest advantage of aproximal analytical solution is a possibility to get analytical solution of PDE. The disadvantages include limited concentration interval of reactive components, not applicable to biosensors with complex biocatalytical schemes, very complex solution of non stationary state, lack of analytical solution for complex initial and boundary conditions.

In the second part of the book the corresponding reaction–diffusion problems are solved using digital modeling. The solving PDE was performed using the finite difference technique. First chapter of Part II covers mathematical models with nonlinear reaction kinetics. The biosensors are assumed to be flat electrodes having a mono-layer of an enzyme applied onto the electrode surface. Coupling the enzyme-catalyzed reaction in the enzyme layer (enzyme membrane) with the one-dimensional-in-space diffusion, the mathematical models are described by the non-stationary reaction-diffusion equations. The biosensors based on amperometric as well as potentiometric transducers are considered. The batch and the injection modes of the biosensor operation are modeled in this chapter. The biosensors utilizing the amplification by the conjugated electrochemical and the enzymatic substrates conversion are also investigated. This chapter ends with the modeling of the biosensors with the substrate as well as the product inhibition. The initial boundary value problems are solved numerically by applying the finite difference technique.

Second chapter of Part II deals with the mathematical models of two types of amperometric multi-enzyme biosensors. One type of the biosensors utilizes enzymatic reactions assuming no interaction between the analyzed substrates and the reaction products. The mathematical model of such biosensors is to simulate the biosensor response to a mixture of compounds (substrates). The second type of the biosensors utilizes the enzymatic reaction followed by a cyclic product conversion. Two kinds of the product regeneration in the two-enzyme biosensors are analyzed: enzymatic and electrochemical.

Third chapter of Part II covers multi-layer mathematical models. The biosensors acting in slightly-stirred buffer solutions are described by two-compartment mathematical models. The biosensor operation is analyzed with a special emphasis to the Nernst diffusion layer. This chapter also discusses the multienzyme systems, were the enzymes are immobilized separately in different active layers packed in a sandwich like multi-layer arrangement. The effect of the diffusion limitation to the substrate is investigated when inert outer membranes are applied to stabilize the enzyme layer and to prolong the calibration curve of the biosensor. This chapter also presents the mathematical models of the amperometric biosensor based on the chemically modified electrode as well as of the peroxidase-based optical biosensor.

Fourth chapter of Part II considers modeling of biosensors for which a two-dimensional-in space domain is required to describe mathematically the biosensor action. Firstly, an amperometric biosensor based on a carbon paste electrode encrusted with a single microreactor is considered. Then, an analytical system based on an array of enzyme microreactors immobilized on a single electrode is investigated. Carbon paste porous electrodes are also investigated by applying a plate-gap model. The last section of the this chapter focuses on the modeling of a practical amperometric biosensor containing the selective and the perforated membranes. The perforated membrane is analyzed with a special emphasis to the geometry of the membrane perforation.

Contemporary numerical methods for solving problems of the mathematical chemistry are gaining increasing popularity. The aim of first chapter of Part III is to introduce the reader with the relevant facts about the basic concepts of the theory of the difference schemes for the linear diffusion equations. The linear diffusion equations play an important and crucial role in most models of a biosensor theory. The most popular simple and together effective difference schemes for the linear diffusion equations are presented here. This method is being frequently used in solving applied problems not only by professional mathematicians, but also by laymen. The concepts presented below are of a primary nature and are sufficient for the solution of the problems of the biosensor. In this book the notations of [222] are mainly applied. The many aspects of the numerical methods for the solution of the partial differential equations are presented in [5, 12, 187, 216].

The difference schemes are extensively applied to the solution of a biosensor problems in second chapter of Part III. This chapter is devoted to various difference approximations of the reaction–diffusion equations. The difference technique, developed in a previous chapter, is employed for the construction of the difference schemes. The main subject of investigation is the system of two nonlinear reaction–diffusion equations in one and two dimensional in space cases.