MODULE DESCRIPTION

| Module title | Module code |
| :--- | :---: |
| Numerical Analysis |  |


| Lecturer(s) | Department where the module is delivered |
| :--- | :--- |
| Coordinator: assoc. prof. dr. Olga Stikonienė | Department of Differential Equations and Numerical <br> Mathematics <br> Other lecturers: |
| Faculty of Mathematics and Informatics <br> Vilnius University |  |


| Cycle | Type of the module |
| :---: | :---: |
| First | Optional |


| Mode of delivery | Semester or period when the <br> module is delivered | Language of instruction |
| :---: | :---: | :---: |
| Face-to-face | Autumn semester <br> Third or Fourth year of study | Lithuanian |

## Prerequisites

Prerequisites: Mathematics for Software Engineering I and II

| Number of credits <br> allocated | Student's workload | Contact hours | Self-study hours |
| :---: | :---: | :---: | :---: |
| 5 | 130 | 68 | 62 |


| Purpose of the module: programme competences to be developed |  |  |
| :---: | :---: | :---: |
| Generic competences: <br> - Communication and collaboration (GK1). <br> Specific competences: <br> - Knowledge and skills of underlying conceptual basis (SK4). |  |  |
| Learning outcomes of the module: students will be able to | Teaching and learning methods | Assessment methods |
| Define and illustrate main concepts related to Numerical Analysis. <br> Formulate and prove main propositions related to Numerical Analysis. | Lecture <br> Practicals with computer Individual reading | Exam (written) Presentation of laboratory work (orally with computer) |
| Apply propositions of Numerical Analysis to solve standard problems analytically and/or using computer software. |  |  |
| Formulate practical problems in mathematical language. |  |  |
| Solve and analyze mathematical models, give conclusions based on mathematical models and justify them logically. |  |  |


| Content: breakdown of the topics | Contact hours |  |  |  |  |  |  | Self-study work: time and assignments |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \mathscr{0} \\ & 0 \\ & U \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { 霛 } \\ & 0 \\ & 0 \end{aligned}$ |  |  | $\begin{aligned} & \text { sun } \\ & \text { 号 } \\ & 3 \\ & 3 \\ & 0.0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 3 \\ & 30 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \tilde{y y} \\ & 0 \\ & 0 \end{aligned}$ | n 0 0 0 0 0 0 | Assignments |
| 1. Computer arithmetic and algorithms | 2 |  |  |  |  |  | 2 | 2 |  |
| 2. Direct methods for solving linear systems. Gauss method. Pivoting technique. Complexity of Gauss method. Tridiagonal systems (Thomas algorithm). Cholesky decomposition method. | 3 |  |  |  | 4 | 1 | 8 | 10 |  |
| 3. Data approximation. Interpolation of functions. Linear and quadratic interpolation. Finite difference quotient formulas and finite difference tables. Newton's interpolating polynomial. Spline interpolation. Linear, quadratic and cubic splines. Least squares method. | 6 |  |  |  | 6 | 1 | 12 | 10 |  |
| 4. Solution of nonlinear equations. Contraction mapping and its properties. Vector norm, matrix norm. Norm of symmetric matrix. Some inequalities for symmetric matrices. Bisection method. Separation of roots. Fixed point method. Newton's method. Method of secants. Solution of nonlinear systems. | 3 |  |  |  | 6 | 1 | 10 | 8 |  |
| 5. Iterative methods for linear systems. Jacobi, Gauss-Seidel, relaxation methods. Implicit stationary iterative methods. Classification of iterative methods. Sufficient condition for convergence of stationary methods. Optimal value of iterative parameter. Condition number. Variational methods: steepest descent, conjugate gradient methods. | 4 |  |  |  | 6 | 1 | 10 | 10 | Individual reading Problem solving 6 laboratory works |
| 6. Numerical integration and ordinary differential equations (ODE). Rectangular formula, trapezoidal formula, Simpson's formula. Methods for estimation of error. Runge's rule. Adaptive methods of numerical integration. Gaussian quadrature. <br> Main concepts of ODE. Euler methods. RungeKutta methods. Truncation error, convergence, consistency and stability. | 6 |  |  |  | 6 | 1 | 10 | 8 |  |
| 7. Eigenvalue problem. Eigenvalues and eigenvectors of matrices. Complete system of vectors. Gram-Schmidt algorithm. Recursive formula for values of characteristic polynomial of tridiagonal systems. Muller's method. Sturm chain and Gershgorin theorem. Power method. Inverse iterations method. Hausholder transformation. | 6 |  |  |  | 4 | 1 | 10 | 10 |  |
| 8. Function optimization methods. Golden section search. Newton's method. Minimization of functions of several variables. Simplex method. Gradient methods. | 2 |  |  |  |  |  | 2 | 4 |  |
| Exam (written) |  | 2 |  |  |  |  | 4 |  | 2 hours for tutorial before exam, 2 hours for exam. |


| Total | $\mathbf{3 2}$ | $\mathbf{2}$ |  |  | $\mathbf{3 2}$ | 6 | $\mathbf{6 8}$ | 62 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| Assessment strategy | Weig <br> ht \% | Deadline | Assessment criteria |
| :--- | :--- | :--- | :--- |
| Practicals with computer | 50 | During the <br> semester | In the laboratory work problems are being solved (with the <br> programs like Maple). During the laboratory work in the <br> computer class students have to demonstrate and explain their <br> solution. <br> Maximum number of points is given for full and correct <br> solution; some points are given for partial solutions. Then <br> average number of points is computed and normalized to 10, <br> where 10 corresponds to the average number of points <br> (laboratory works) of the best student. <br> These points constitute 50\% of the final marks. |
| Exam (written) | 50 | Exam session | All questions are worth the same number of points. Maximal <br> number of points is given if the student answered the question: <br> the student has given correct definitions, has given correct <br> statements and their proofs. Some points are given for partial <br> answers. Then average number of points is computed and <br> normalized to 10, where 10 correspond to the average number <br> of points of the best student. <br> These points constitute 50\% of the final mark |


| Author | Publis hing year | Title | Number or volume | Publisher or URL |
| :---: | :---: | :---: | :---: | :---: |
| Required reading |  |  |  |  |
| V.Būda, R.Čiegis | 1997 | Computational mathematics (in Lithuanian) |  | Vilnius: TEV |
| O. Štikonienė |  | Numerical Analysis. Lecture notes (in Lithuanian) |  | http://www.mif.vu.lt/~olgas/S M.html |
| Recommended reading |  |  |  |  |
| A.Quarteroni, F.Saleri and <br> P. Gervasio | 2010 | Scientific Computing with MATLAB and Octave |  | Springer |
| J.H.Mathews, K.D.Fink | 2004 | Numerical methods Using MATLAB |  | Prentice Hall http://math.fullerton.edu/mathe ws/numerical.html |
| B.Kvedaras, M.Sapagovas | 1974 | Numerical Analysis (in Lithuanian) |  | Vilnius: Mintis |
| R.Čiegis | 2003 | Numerical methods for differential equations (in Lithuanian) |  | Vilnius: Technika |
| A.Quarteroni, R.Sacco, F.Saleri | 2000 | Numerical Mathematics |  | Springer |
| V.Būda, M.Sapagovas | 1998 | Numerical methods. Algorithms, problems, projects (in Lithuanian) |  | Vilnius: Technika |
| K. Plukas | 2001 | Numerical methods and algorithms (in Lithuanian) |  | Kaunas: N. lankas |

