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9th School Olympiad of Lithuania for youngsters 2007

associated with the 22nd Lithuanian team-contest

Grades 7 and 8

Department for Mathematics and Informatics of Vilnius University
September the 29th 2007

1 **(A)** Baron Munchhausen deeply believes that it is possible to indicate such 4 distinct 4-digit positive integers consisting only of digits 1, 2 and 3 such that any two of these numbers have equal digits in at most one position. Is it really so? Could you ever indicate for him such 4 positive integers.

(B) Baron Munchhausen never thinks that it is possible to indicate such 6 distinct 4-digit positive integers consisting only of digits 1, 2 and 3 such that any two of these numbers have equal digits in at most one position. Is it really so? Could you ever indicate such 6 numbers..

(C) Find the maximum number of distinct 4-digit positive integers consisting only of digits 1, 2 and 3 such that any two of these numbers have equal digits in at most one position.

2. **(A)** Baron Munchhausen claims that it is impossible to arrange all integers 1 to 16 on a straight line so that the sum of any two adjacent numbers is the square of an integer. Is it indeed so?

(B) Baron Munchhausen claims that it is easily possible to arrange all integers 1 to 16 on a circle so that the sum of any two adjacent numbers is the square of an integer. Is it indeed so?

3. Points K and L are taken by Winnie-the-Pooh on the sides BC and CD of a square $ABCD$ so that $\angle AKB = \angle AKL$. Help Winnie to indicate the true magnitude of $\angle KAL$.

4. **(A)** Mr Sherlock Holmes together with Dr Watson wish to find all such a pairs (x, y) of positive integers x and y such that

$$x^2 - y^2 - x + y = 10.$$

How many and what pairs they will find?

(B) Help them by their attempts if only possible to indicate a pair (x, y) of positive integers x and y such that

$$x^2 - y^2 - x + y = 2007.$$

5. A square consists of 7×7 identical quadratic squares. Some of them Winnie-the-Pooh had coloured black in such a way that numbers of black squares in each row and in each column are even (possibly 0).

(A) Is it possible for Winnie to colour exactly 4 quadratic squares in such a way that the given condition is satisfied?

(B) Is it possible for Winnie to colour exactly 6 quadratic squares in such a way that the given condition is satisfied?

(C) What number of quadratic squares would be possible for Winnie to colour in that way? Indicate all possible cases.