**4th Mathematical Contest of Friendship
in Honor and Memory of Grand Duchy of Lithuania**

30th September 2012

**Welcomed by Professor Nazar Agakhanov, the Chairman
of the Advisory Board of the International Mathematical Olympiad**

1. Find all functions $g : \mathbb{R} \to \mathbb{R}$, for which there exists a strictly increasing function $f : \mathbb{R} \to \mathbb{R}$ such that
$$f(x + y) = f(x)g(y) + f(y).$$

2. The base AB of a trapezium $ABCD$ is longer than the base CD, and $\angle ADC$ is a right angle. The diagonals AC and BD are perpendicular. Let E be the foot of the altitude from D to the line BC. Prove that
$$\frac{AE}{BE} = \frac{AC \cdot CD}{AC^2 - CD^2}.$$

3. How many ways are there to line up 19 girls (all of different heights) in a row so that no girl has a shorter girl both in front of and behind her?

4. Let m be a positive integer. Find all bounded sequences of integers $a_1, a_2, a_3, \ldots$ for which $a_n + a_{n+1} + a_{n+m} = 0$ for all $n \in \mathbb{N}$.