3rd MATHEMATICAL CONTEST
of Friendship
in Honor and Memory
OF GRAND DUCHY OF LITHUANIA

1. Integers a, b and c satisfy the condition $ab + bc + ca = 1$. Is it true that the number $(1+a^2)(1+b^2)(1+c^2)$ is a perfect square? Why?

2. Let $n \geq 2$ be a natural number and suppose that positive numbers $a_0, a_1, ..., a_n$ satisfy the equality
$$(a_{k-1} + a_k)(a_k + a_{k+1}) = a_{k-1} - a_{k+1} \text{ for each } k = 1,2,...,n-1.$$
Prove that $a_n < \frac{1}{n-1}$.

3. Find all primes p, q such that $p^3 - q^7 = p - q$.

4. In the cyclic quadrilateral *ABCD* with *AB* = *AD* points *M* and *N* lie on the sides *CD* and *BC* respectively so that *MN* = *BN* + *DM*. Lines *AM* and *AN* meet the circumcircle of *ABCD* again at points *P* and *Q* respectively. Prove that the orthocenter of the triangle *APQ* lies on the segment *MN*.

5. Positive integers 1, 2, 3, ..., n are written on a blackboard ($n > 2$). Every minute two numbers are erased and the least prime divisor of their sum is written. In the end only the number 97 remains. Find the least n for which it is possible.