ON THE CENTRAL LIMIT THEOREM FOR MULTIPARAMETER STOCHASTIC PROCESSES

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l.INTRODUCTION AND RESULTS In recent papers Bezandry and Fernique (1990,1992), Fernique (1993) have given new convergence and tightness criteria for random processes whose sample paths are right-continuous and have left-limits. These criteria have been applied by Bezandry and Fernique, Bloznelis and Paulauskas to prove the central limit theorem (CLT) in the Skorohod space $D[0,1]$.

In this paper, using recent technique of Bezandry and Fernique, we improve some results of Bickel and Wichura (1971) on weak convergence and tightness for multiparameter processes. The main results of the paper deals with stochastically continuous processes and may be viewed as an extension to multidimensional case of the weak convergence criteria due to Bezandry and Fernique (1990,1992) and of the CLT due to Bloznelis and Paulauskas (1993), Fernique (1993).

Let $X, X_1, X_2, ...$ be i.i.d. random processes with sample paths in Skorohod space $D_k \equiv D([0,1]^k, R)$. For details about the space D_k endowed with the Skorohod topology we refer to Neuhaus (1971) and Straf (1972). Denote $S_n = n^{-1/2}(X_1 + \ldots + X_n - n\mathbf{E}X)$. A random process X is said to satisfy the CLT in D_k $\quad (X \in CLT(D_k))$ if the distributions of S_n converge weakly to a Gaussian distribution on D_k. For a random process $X = \{X(t), t \in [0,1]^k\}, k \geq 1$ define

$$\mathbf{\Delta}^{(i)}_{(a,b]} X(u) = X(u_1, \ldots u_{i-1}, b, u_{i+1}, \ldots, u_k) - X(u_1, ..., u_{i-1}, a, u_{i+1}, ..., u_k),$$

$$\mathbf{u} = (u_1, ..., u_k) \in [0,1]^k, \qquad 1 \leq i \leq k, \quad a, b \in [0,1].$$

A rectangle B in the unit cube $T \equiv [0,1]^k$ is a subset of T of the form

$$(\mathbf{s}, \mathbf{t}] = \prod_{i=1}^{k} (s_i, t_i], \quad \mathbf{s} = (s_1, ..., s_k), \quad \mathbf{t} = (t_1, ..., t_k) \in T;$$

the i-th face of B is $\prod_{j\neq i}(s_j,t_j]$. Disjoint rectangles B and C are neighbours if they abut and have the same i-th face for some i. For a rectangle B=($\mathbf{s}$,$\mathbf{t}$] let

$$X(B)=\mathbf{\Delta}^{(1)}_{(s_1,t_1]}\dots\mathbf{\Delta}^{(k)}_{(s_k,t_k]}X(\mathbf{u})$$

be the increment of X around B; $X(\cdot)$ is a random finitely additive function on rectangles. The set $LB(T)=\{(t_1,...,t_k)\in T: t_i=0 \quad \texttt{for some} \quad i\}$ is called the lower boundary of T, and $UB(T)=\{(t_1,...,t_k)\in T: t_i=1 \quad \texttt{for some} \quad i\}$ is called the upper boundary of T.

THEOREM 1. Let $p,q\geq 2$ and $k\geq 1$. Let $X=\{X(t), \quad t\in T\}$ be a random process with $EX(t)=0, \quad EX^2(t)<\infty$ for each $t\in T$. Assume X vanishes along the lower boundary of T, i.e.,

$$P(\quad X(t)=0 \quad) \;=\; 1 \qquad \texttt{for all} \quad t\in LB(T). \tag{1.1}$$

Assume there exist nondecreasing non-negative functions f,g and finite measures F,G on T with continuous marginals such that for all neighbouring rectangles $B,C\subset T$

$$\mathbf{E}(\mid X(B)\mid\wedge\mid X(C)\mid)^p\leq f(F(B\cup C)), \tag{1.2}$$

$$E\mid X(B)\mid^q\leq g(G(B)) \tag{1.3}$$

and for some $\varepsilon>0$

$$\int_0^{\varepsilon}(u)^{-1-1/p}f^{1/p}(u)log^{k-1}(u^{-1})du<\infty, \tag{1.4}$$

$$\int_0^{\varepsilon}(u)^{-1-1/(2q)}g^{1/q}(u)log^{k-1}(u^{-1})du<\infty. \tag{1.5}$$

Then X has a version X' with sample paths in D_k and $X'\in CLT(D_k)$.

Note that the random process X is stochastically continuous by (1.3), (1.5). For one-parameter processes ($k=1$) Theorem 1 coincides with Th.2 of Bloznelis and Paulauskas (1993b), see also Fernique (1993). Condition (1.1) which appeared yet in Chentsov (1956) and Bickel and Wichura (1971) (and is restrictive for $k\geq 2$)

could be explained by the fact that (1.2) and (1.3) do not control behaviour of X on the boundary of T because the measures F and G have continuous marginals. This condition may be removed if one requires X to satisfy (1.2), (1.3) on each coordinate cube laying on the lower boundary of T, namely, if for each number $m < k$, each collection $\{i_1, ..., i_m\} \subset \{1, ..., k\}$ and each m-dimensional unit cube

$$T(i_1, ..., i_m) = \{(t_1, ..., t_k) \in T : t_i = 0 \quad for \quad i \not\in \{i_1, ..., i_m\}\}$$

there exist functions f, g, satisfying (1.4), (1.5) with the parameter $(k =)m$ and measures F, G on $T(i_1, ..., i_m)$ with continuous marginals such that for all neighbouring rectangles $B, C \subset T(i_1, ..., i_m)$ conditions (1.2), (1.3) are satisfied, c.f., Lachout (l988).

As in the case $k = 1$ conditions (1.2), (1.4) are optimal in the following sense.

EXAMPLE 1. Let f be nondecreasing positive function on [0,1] satisfying the following two conditions:

$$\int_0^1 \cdots \int_0^1 \big((u_1 \cdots u_k)^{-1} f(u_1 \cdots u_k)\big)^{1/p} (u_1 \cdots u_k)^{-1} du_1 \cdots du_k = \infty; \qquad (1.6)$$

there exist positive constants K and α such that for all $0 < x < y \leq 1$

$$x^{-\alpha} f(x) \leq K y^{-\alpha} f(y). \qquad (1.7)$$

Then there exists a stochastically continuous process X with sample paths in D_k such that (1.2) and an analogous condition on the LB(T) (which was discussed above) are satisfied but $X \notin CLT(D_k)$.

In the proof of Theorem 1 we use a general statement about the desymmetrization and weak convergence of sequences of stochastically continuous processes viewed as D_k valued random elements, cf. Bloznelis and Paulauskas (1993) and Fernique (1993).

For the sake of completeness we recall some basic facts about the space D_k , see , e.g., Bickel and Wichura (1971) (we will abbreviate this reference by [B & W] in the sequel). Functions $\mathbf{x} : [0,1]^k \to R$ in D_k may be characterized by their continuity properties as follows: if $\mathbf{t} \in T$ and if for $1 \leq i \leq k$, R_i is one of the relations $<$ and $\geq$, let $Q = Q(R_1, ..., R_k, \mathbf{t})$ denote the quadrant $\{(s_1, ..., s_k) \in T : s_i R_i t_i, \quad 1 \leq i \leq k\}$. Then $\mathbf{x} \in D_k$ iff for each $\mathbf{t} \in T$,

$$\mathbf{x}_Q \equiv \lim_{\mathbf{s} \to \mathbf{t}, \mathbf{s} \in Q} \mathbf{x}(\mathbf{s})$$

exists for each quadrant Q, and $\mathbf{x}(t) = \mathbf{x}_{Q(\geq,...,\geq,\mathbf{t})}$. In this sense, the functions of D_k are "continuous from above, with limits from below". One can define Skorohod distance between $\mathbf{x}$ and $\mathbf{y}$ in D_k to be

$$d(\mathbf{x}, \mathbf{y}) = \inf\{\max\{\sup_{\mathbf{t} \in T} \mid \mathbf{x}(\mathbf{t}) - \mathbf{y}(\lambda(\mathbf{t})) \mid, \sup_{\mathbf{t} \in T} \|\lambda(\mathbf{t}) - \mathbf{t}\|\}\},$$

where the infimum is taken over all hoemorphisms $\lambda : T \to T$,

$$\lambda(t_1, ..., t_k) = (\lambda_1(t_1), ..., \lambda_k(t_k)); \lambda_i(0) = 0, \lambda_i(1) = 1,$$

λ_i is increasing continuous, $1 \leq i \leq k$ and $\|\mathbf{s} - \mathbf{t}\| = \max_i \mid s_i - t_i \mid$ for $\mathbf{s}, \mathbf{t} \in T$. Note that D_k coincides with the usual Skorohod space $D[0, 1]$ when $k = 1$.

Let $(\Omega, \mathcal{F}, P)$ denote a probability space on which the random elements under consideration will be defined. We say that a random process $X = \{X(t), t \in T\}$ satisfies *condition* (A) on the unit cube T if there exist positive nondecreasing functions $f_1, ..., f_M$, $\theta_1, ..., \theta_M$, additionally θ_i is convex, $1 \leq i \leq M$, and measures $F_1, ..., F_M$ on T with continuous marginal distributions such that for all neighbouring rectangles $B, C \subset T$ and each $A \in \mathcal{F}$

$$E\{(\mid X(B) \mid \wedge \mid X(C) \mid) \mathbb{I}_A\} \leq \sum_{i=1}^{M} f_i(F_i(B \cup C)) \theta_i(P(A)) \tag{1.8}$$

and functions f_i, θ_i satisfy

$$\int_0^{\varepsilon} (u)^{-2} log^{k-1}(1 + u^{-1}) \sum_{i=1}^{M} f_i(u) \theta_i(u) du < \infty. \tag{1.9}$$

$Condition(A)$ is a multidimensional analogue of the corresponding one formulated in Bezandry and Fernique (1992) (for brevity we use [B & F] in the sequel). We say that X satisfies *condition*(B) if for each $m < k$ and each $\{i_1, ..., i_m\} \subset \{1, ..., k\}$ X satisfies *condition*(A) on the cube $T(i_1, ..., i_m)$. Let $\{X_n, n \geq 1\}$ be a sequence of random processes defined on T. We say that the sequence $\{X_n, n \geq 1\}$ satisfies *condition*(A) if each X_n satisfies (A) with the same functions f_i, θ_i and measures $F_i, 1 \leq i \leq M$.

Continuity properties of sample paths. The next theorem gives sufficient conditions for a random process X to have sample paths in D_k.

THEOREM 2. Let $X = \{X(t), t \in [0,1]^k\}$ be a random process satisfying conditions (1.1) and (A). Suppose that for all $\varepsilon > 0$

(*i*) $P(\mid X(t+h) - X(t) \mid > \varepsilon) \to 0$ as h tends to 0 "from above";

(*ii*)

$P(\mid \mathbf{\Delta}^{(i)}_{(a,1]} X(u) \mid > \varepsilon) \to 0$ as $a \to 1$, for all $i \in \{1, ..., k\}$ and all $u \in UB(T)$.

Then X has a version with sample paths in D_k.

Theorem 2 is an extension to multiparameter case of Theorem 1.2 of [B & F], cf. also Theorem 4 of Bickel and Wichura (1971). Note that a stochastically continuous random process X always satisfies conditions (i) and (ii) and *condition*(A) is fulfiled if X satisfies (1.2), (1.4). Statement of Theorem 2 remains true if one replace condition (1.1) by the weaker condition (B).

COROLLARY 3. Let $X = \{X(t), t \in T\}$ be a stochastically conti nuous random process satisfying (1.1) and there exist nondecreasing non-negative function f satisfying (1.4) and measure F on T with continuous marginals such that (1.2) is fulfilled. Then X has a version with sample paths in D_k.

Let us compare condition (1.2), (1.4) with the corresponding one of Theorem 4 of [B & W] :

$$\exists \alpha, \gamma > 0 : \forall \varepsilon > 0, P(\mid X(B) \mid \wedge \mid X(C) \mid > \varepsilon) < \varepsilon^{-\gamma}(F(B \cup C))^{1+\alpha}. \tag{1.10}$$

If

$$E(\mid X(B) \mid \wedge \mid X(C) \mid)^{\gamma} \leq (F(B \cup C))^{1+\alpha}. \tag{1.11}$$

then one may check (1.10) using Tchebyshev inequality, see ,e.g., Theorem 6 ibidem. If $\gamma \geq 2$, then the condition (1.11) implies (1.2) ,(1.4).

Weak convergence. A random process X with sample paths in D_k is said to be continuous at the upper boundary of T if for each $i \leq k$

$$\lim_{a \uparrow 1} \sup_{s \in [0,1]^k} \mathbf{\Delta}^{(i)}_{(a,1]} X(s) = 0 \texttt{ with probability } 1. \tag{1.12}$$

Let $\mathcal{U}$ be a collection of subsets of T of the form $\mathcal{U} = U_1 \times ... \times U_k$, where each $U_i \subset [0,1]$ contains zero and one and has countable complement.

THEOREM 4 (cf. Th.1.3. in [B & F]). Let $X_n, n \geq 1$ and X be random processes with sample paths in D_k and suppose that X is continuous at the upper boundary of T. Assume the sequence $\{X_n, n \geq 1\}$ satisfies *condition*(A) and each X_n satisfies (1.1). If for some $U \in \mathcal{U}$ and all choises $t^1, ..., t^r \in U$

$$(X_n(t^1), ..., X_n(t^r)) \longrightarrow^{\mathcal{D}}_{n \to \infty} (X(t^1), ..., X(t^r)), \tag{1.13}$$

converge in distribution, then $X_n \Rightarrow X$ (converge weakly in D_k).

Condition (1.12) appears in [B & W] and may be viewed as a multidimensional analogue of the condition $P(X(1) \neq \lim_{t \uparrow 1} X(t)) = 0$ of Theorems 15.4, 15.6 of Billingsley (1968). Denote

$$\omega_\delta(x) = \sup\{| x(s) - x(t) | : s, t \in T, \|s - t\| < \delta\}, x \in D_k.$$

LEMMA 5 (see, e.g., Neuhaus (1971)). Let $\{X_n, n \geq 1\}$ be a sequence of random processes with sample paths in D_k. Let Y be a continuous random process on $[0,1]^k$. Assume that $X_n \Rightarrow Y$ converge weakly in D_k. Then

$$\forall \varepsilon > 0, \forall \eta > 0 \quad \exists \delta > 0 \quad \exists n_\circ : \quad P(\omega_\delta(X_n) > \varepsilon) < \eta, \quad \forall n > n_\circ.$$

LEMMA 6 (cf. Lemma 2 in Bloznelis and Paulauskas (1993) and Lemma 2.3 in Fernique (1993)). Let $\{X_n, n \geq 1\}$ be a sequence of stochastically continuous random processes with sample paths in D_k and $\overline{X}_n$ be an independent copy of

$X_n, n \geq 1$. Assume there exists a random process X with sample paths in D_k such that (1.13) is satisfied. Assume $\{X_n^* = X_n - \overline{X}_n, n \geq 1\}$ converge weakly in D_k to sample continuous random process. If, moreover, the sequence $\{X_n, n \geq 1\}$ is uniformly stochastically continuous:

$$\forall \varepsilon, \eta > 0 \; \exists \delta > 0 : \|t - s\| < \delta \;\Rightarrow\; P(\mid X_n(t) - X_n(s) \mid > \varepsilon) < \eta, \; n \geq 1,$$

then $X_n \Rightarrow X$ converge weakly and X is sample continuous.

For $k = 1$ these lemmas are proved in Bloznelis and Paulauskas (1993). Argument used there easily extends to multidimensional case.

PROPOSITION 7. Let $X = \{X(t), \; t \in [0,1]^k\}$ be a centered random process with sample paths in $D[0,1]^k$. Assume X satisfies CLT in $D[0,1]^k$ and the limiting Gaussian r. process Y is sample continuous. Then the sequence of processes

$$\{Z_n(s,t) = \sum_{i \leq ns} n^{-1/2} X_i(t), \; s \in [0,1], t \in [0,1]^k\}, \; n \geq 1$$

converge weakly in $D[0,1]^{k+1}$ to the Brownian motion W_Y in $C[0,1]^k$, generated by Y.

We recall that $W_Y = W_Y(s, \mathbf{t})$ is defined on $[0,1]^{k+1}$ and has covariance

$$Cov[(W_Y(s', \mathbf{t}'), W_Y(s'', \mathbf{t}'')] = \min(s', s'') \times Cov(X(\mathbf{t}'), X(\mathbf{t}'')).$$

The equivalence of the CLT for (separable) Banach space valued random elements and the Invariance principle was proved by Kuelbs (1973). But the space D_k considered in Proposition 7 is not a topological linear space.

2. PROOFS

Let $F_i, 1 \leq i \leq M$ be finite measures on $[0,1]^k$ with continuous marginals. Denote by λ the Lebesgue measure on $[0,1]^k$ and define the measure $F' = c(F_1 + ... + F_M + \lambda)$, where c is the norming constant which makes F' to be a probability measure. It is easy to see that (1.6) remains true if one replace $f_i(F_i(B \cup C))$ in the right-hand side by $f_i'(F'(B \cup C)), 1 \le i \le M$, where the functions $f_i'(u) = f_i(c^{-1}u)$ satisfy (1.7) as well as $f_i, 1 \le i \le M$. In what follows we assume without loss of generality that $F_1 = ... = F_M := F$, and F is a probability measure with continuous and strictly increasing marginal distribution functions (m.d.f.).

Recall some useful notation from [B & W]. Let $x : [0,1]^k \to R$. For each $i \in \{1, ..., k\}$ and each $t \in [0,1]$ define

$$x_{(t)}^{(i)} : [0,1]^{i-1} \times [0,1]^{k-i} \to R$$

by

$$x_{(t)}^{(i)}(t_1, ..., t_{i-1}, t_{i+1}, ..., t_k) = x(t_1, ..., t_{i-1}, t, t_{i+1}, ..., t_k)$$

and for each $0 \le s \le t \le u \le 1$ set

$$\Delta(s,t,u)(x^{(i)}) = \|x_{(t)}^{(i)} - x_{(s)}^{(i)}\| \wedge \|x_{(u)}^{(i)} - x_{(t)}^{(i)}\|,$$

where $\|x_{(t)}^{(i)}\| = \sup\{| \, x(t_1, .., t_{i-1}, t, t_{i+1}, ..., t_k) \, | : \ 0 \le t_j \le 1, \ j \ne i\}$. Define the modulus

$$\overline{\omega}_\delta''(x) = \sum_{i=1}^{k} \omega_\delta''(x^{(i)}),$$

where

$$\omega_\delta''(x^{(i)}) = \sup\{\Delta(s,t,u)(x^{(i)}) \ : \ 0 \le s \le t \le u \le 1, \ u - s \le \delta\}.$$

Then the following inequality is true, see [B & W],

$$\|x\| \le \overline{\omega}_1''(x) + | \, x(1,1,...,1) \, |, \tag{2.1}$$

provided x vanishes on the $LB(T)$. Here $\|x\| = \sup\{| \, x(t) \, |, t \in T\}$.

LEMMA. Let $k \ge 1$. Let X be a random process with sample paths in D_k and X satisfies (1.1). Assume X satisfies *condition*(A). Then

$$\forall \varepsilon, \eta > 0 \ \exists \delta > 0 \ : \ P(\overline{\omega}_\delta''(X) > \varepsilon) < \eta. \tag{2.2}$$

Here $\delta = \delta(\varepsilon, \eta, (F_i, f_i, \theta_i, 1 \le i \le M))$.

Proof. Without loss of generality we assume that $F_1 = F_2 = ... = F_M := F$ and F is a probability measure with continuous and strictly increasing marginal distribution functions. By the set-theoretic identity

$$D([0,1]^k,\ R) \ = \ D([0,1], D_{k-1})$$

which is valid via any one of the correspondences $x(.) \leftrightarrow x^{(i)}_{(.)}(.),\ 1 \leq i \leq k$, provided on the right-hand side D_{k-1} is equiped with the supremum norm, one reduce the problem to the one-parameter case, see [B & W]. Then for one-parameter and functional space D_{k-1} valued process $X^{(i)}_{(t)}(.),\ t \in [0,1]$ the scheme of approximation from [B & F] , Lemma 1.1.3 , is applied. To prove the lemma we show that

$$\mathbf{E}\omega''_{\delta}(X^{(i)}) = o(1),\ \ \texttt{as}\ \ \delta \to 0\ \texttt{for}\ \ 1 \leq i \leq k.$$

For the sake of simplicity we consider the case of 2-dimensional time ($k = 2$). The same proof carries over for general k.

Let F_i denotes i-th marginal distribution function of F. Fix $i = 1$. For the one-parameter random process $X^{(1)}$ with values in D_1 (in D_{k-1} for general k) we use approximation as in Lemma 1.1.3 of [B & F] but with the discretization by means of the sets

$$S_m = \{t = F_1^{-1}(j \cdot 2^{-m}),\ 0 \leq j \leq 2^m\},\ m \geq 0.$$

Let t^-_m and t^+_m in S_m be the two nearest neighbours of t satisfying $t^-_m < t < t^+_m,\ t \in (0,1)$. Let $m \geq 0$ and denote

$$\delta_m = \min\{|\ s - t\ |,\ s,t \in S_m,\ s \neq t\};$$

$$A_{m,s} = \{\omega \in \Omega : \sup_{t \in S_{m+1} \setminus S_m} \Delta(t^-_m, t, t^+_m)(X^{(1)}) = \Delta(s^-_m, s, s^+_m)\},\ s \in S_{m+1} \setminus S_m\ ;$$

$$B_{m,s} = \{\omega \in \Omega : \sup_{t \in S_m \setminus \{0;1\}} \Delta(t^-_m, t, t^+_m) = \Delta(s^-_m, s, s^+_m)\}, s \in S_m \setminus \{0;1\}.$$

We may assume that

$$A_{m,s} \cap A_{m,t} \ = \ \emptyset \ \ \texttt{if} \ \ s \neq t. \tag{2.3}$$

Indeed, if (2.3) is not satisfied one may put $\overline{A}_{m,s} = A_{m,s} \setminus (\bigcup_{t<s} A_{m,t})$ instead of $A_{m,s}$. Analogously we assume that $B_{m,s} \cap B_{m,t} = \emptyset$ if $s \neq t$. We have, see the proof of Theorem 1.2 and Lemma 1.3.1 in [B & F], that

$$\omega''_{\delta_{r+1}}(X^{(1)}) \leq \sup_{t \in S_r \setminus \{0;1\}} \Delta(t_r^-, t, t_r^+)(X^{(1)}) + 2 \cdot \sum_{m \geq r} \sup_{t \in S_{m+1} \setminus S_m} \Delta(t_m^-, t, t_m^+)(X^{(1)}) \leq$$

$$\leq \sum_{t \in S_r \setminus \{0;1\}} \Delta(t_r^-, t, t_m^+)(X^{(1)}) \cdot \mathrm{I\!I}_{B_{r,t}} + 2 \sum_{m \geq r} \sum_{t \in S_{m+1} \setminus S_m} \Delta(t_m^-, t, t_m^+)(X^{(1)}) \cdot \mathrm{I\!I}_{A_{m,t}} :=$$

$$= I_1 + I_2. \tag{2.4}$$

Let m and $t \in (0,1)$ be fixed and define the random processes

$$Y_m^{(t)} = X_{(t)}^{(1)} - X_{(t_m^-)}^{(1)}, \; Z_m^{(t)} = X_{(t_m^+)}^{(1)} - X_{(t)}^{(1)}.$$

Processes $Y_m^{(t)}$ and $Z_m^{(t)}$ have sample paths in D_1 (in D_{k-1} for general k) and by (2.1)

$$\Delta(t_m^-, t, t_m^+)(X^{(1)}) \leq \overline{\omega}''_1(Y_m^{(t)}) + \overline{\omega}''_1(Z_m^{(t)}) + \mid Y_m^{(t)}(1) \mid \wedge \mid Z_m^{(t)}(1) \mid,$$

see [B & W]. Note that for one-parameter processes the equality $\overline{\omega}'' = \omega''$ holds. By (2.4)

$$\mathbf{E}\omega''_{\delta_{r+1}}(X^{(1)}) \leq \mathbf{E}I_1(Y) + \mathbf{E}I_1(Z) + \mathbf{E}I_1(Y,Z) + \mathbf{E}I_2(Y) + \mathbf{E}I_2(Z) + \mathbf{E}I_2(Y,Z), \tag{2.5}$$

where

$$I_1(Y) = \sum_{t \in S_r \setminus \{0;1\}} \omega''_1(Y_r^{(t)}) \cdot \mathrm{I\!I}_{B_{r,t}},$$

$$I_2(Y) = 2 \cdot \sum_{m \geq r} \sum_{t \in S_{m+1} \setminus S_m} \omega''_1(Y_m^{(t)}) \cdot \mathrm{I\!I}_{A_{m,t}}, \tag{2.6}$$

$I_1(Z)$ and $I_2(Z)$ are defined analogously;

$$I_1(Y,Z) = \sum_{t \in S_r \setminus \{0;1\}} \mid Y_r^{(t)}(1) \mid \wedge \mid Z_r^{(t)}(1) \mid \cdot \mathrm{I\!I}_{B_{r,t}},$$

$$I_2(Y,Z) = 2 \cdot \sum_{m \geq r} \sum_{t \in S_{m+1} \setminus S_m} \mid Y_m^{(t)}(1) \mid \wedge \mid Z_m^{(t)}(1) \mid \mathrm{I\!I}_{A_{m,t}}.$$

First, estimate $\mathbf{E}I_2(Y,Z)$. For a fixed m and $t \in S_{m+1} \setminus S_m$

$$\mathbf{E} \mid Y_m^{(t)}(1) \mid \wedge \mid Z_m^{(t)}(1) \mid \cdot \mathbb{I}_{A_{m,t}} =$$

$$= \mathbf{E} \mid X((t_m^-, t] \times (0,1]) \mid \wedge \mid X((t, t_m^+] \times (0,1]) \mid \cdot \mathbb{I}_{Am,t} \leq$$

$$\leq \sum_{i=1}^{M} f_i(2^{-m}) \cdot \theta_i(P(A_{m,t})), \tag{2.7}$$

by $condition(A)$, because

$$F((t_m^-, t] \times (0,1] \cup (t, t_m^+] \times (0,1]) = F_1(t_m^+) - F_1(t_m^-) = 2^{-m}.$$

It is easy to show, see Lemma 1.0.3 in [B & F], that for a concave function θ_i

$$\sum_{t \in S_{m+1} \setminus S_m} \theta_i(P(A_{m,t})) \leq 2^m \cdot \theta_i(2^{-m}), \tag{2.8}$$

becasuse of $\sum_t P(A_{m,t}) \leq 1$ by (2.3). Here $2^m = \sharp\{S_{m+1} \setminus S_m\}$. It follows fom (2.7) and (2.8) that

$$\mathbf{E}I_2(Y,Z) \leq 2 \cdot \sum_{m \geq r} \sum_{i=1}^{M} 2^m \cdot f_i(2^{-m}) \cdot \theta_i(2^{-m}).$$

Similarly, we get the estimate

$$\mathbf{E}I_1(Y,Z) \leq \sum_{i=1}^{M} f_i(2^{-(r-1)}) \cdot (2^r - 1) \cdot \theta_i(1/(2^r - 1)) \leq$$

$$\leq 2 \cdot \sum_{i=1}^{M} 2^{(r-1)} \cdot f_i(2^{-(r-1)}) \cdot \theta_i(2^{-(r-1)}).$$

By monotonicity of f_i and θ_i one may estimate the series by the integral

$$\mathbf{E}I_1(Y,Z) + \mathbf{E}I_2(Y,Z) \leq 2 \cdot \int_0^{\tau} \sum_{i=1}^{M} u^{-2} \cdot f_i(u) \cdot \theta_i(u) du, \tag{2.9}$$

where $\tau = 2^{-r+2}$.

Now estimate $\mathbf{E}I_2(Y)$. Fix $m \geq 0$, $t \in (0,1)$. Let $F_{2,t}$ denote the second coordinate marginal distribution of measure F restricted on $(t_m^-, t] \times [0,1]$,

$$F_{2,t}(b) - F_{2,t}(a) = F((t_m^-, t] \times (a,b]),\ 0 \leq a \leq b \leq 1.$$

Let $t \in S_{m+1} \setminus S_m$ and note that $F_{2,t}(1) - F_{2,t}(0) = F_1(t) - F_1(t_m^-) = 2^{-m-1}$. To estimate $\omega_1''(Y_m^{(t)})$ we use discretization as in Lemma 1.1.3 of [B & F] but by means of the sets

$$S_{n,t} = \{s = F_{2,t}^{-1}(j \cdot 2^{-n} \cdot 2^{-m-1}),\ 0 \leq j \leq 2^n\}, \quad n \geq 0.$$

For $s \in (0,1)$ choose s_n^- and s_n^+ in $S_{n,t}$ to be the nearest neighbours satisfying $s_n^- < s < s_n^+$. We argue as in the proof of Theorem 1.2. and of Lemma 1.3.1 of [B & F] to show that

$$\omega_1''(Y_m^{(t)}) \leq 2 \cdot \sum_{n \geq 0} \sum_{s \in S_{n+1,t} \setminus S_{n,t}} \Delta(s_n^-, s, s_n^+)(Y_m^{(t)}) \cdot \mathrm{I\!I}_{D_{n,s}},$$

where

$$D_{n,s} = \{\omega : \sup\{\Delta(v_n^-, v, v_n^+)(Y_m^{(t)}) : v \in S_{n+1,t} \setminus S_{n,t}\}$$
$$= \Delta(s_n^-, s, s_n^+)(Y_m^{(t)})\}, \quad s \in S_{n+1,t} \setminus S_{n,t}$$

and one may assume, that $D_{n,s} \cap D_{n,t} = \emptyset$ if $s \neq t$, cf (2.3). Note also, that

$$\Delta(s_n^-, s, s_n^+)(Y_m^{(t)}) = \mid X((t_m^-, t] \times (s_n^-, s]) \mid \wedge \mid X((t_m^-, t] \times (s, s_m^+]) \mid$$

and $F((t_m^-, t] \times (s_n^-, s_n^+]) = 2^{-m-1} \cdot 2^{-n}$. Hence for each $m \geq 0$ and $t \in S_{m+1} \setminus S_m$

$$\mathbf{E}\omega_1''(Y_m^{(t)}) \cdot \mathrm{I\!I}_{A_{m,t}} \leq 2 \cdot \sum_{n \geq 0} \sum_{s \in S_{n+1,t} \setminus S_{n,t}} (\sum_{i=1}^{M} f_i(2^{-m-1} \cdot 2^{-n}) \cdot \theta_i(P(A_{m,t} \cap D_{n,s}))).$$

We argue as above, see (2.6), to show that

$$\sum_{t \in S_{m+1} \setminus S_m} \sum_{s \in S_{n+1,t} \setminus S_{n,t}} \theta_i(P(A_{m,t} \cap D_{n,s})) \leq 2^n \cdot 2^m \cdot \theta_i(2^{-n} \cdot 2^{-m}).$$

Now we are in position to estimate

$$\mathbf{E}I_2(Y) \leq 2 \cdot \sum_{m \geq r} \sum_{n \geq 0} [\sum_{i=1}^{M} f_i(2^{-n-m-1}) \cdot 2^{n+m} \cdot \theta_i(2^{-n-m})].$$

It is easy to show that the quantity on the right-hand side of the inequality does not exceed

$$8 \int_0^{\tau} \int_0^1 (u_1 u_2)^{-2} [\sum_{i=1}^{M} f_i(u_1 u_2) \cdot \theta(u_1 u_2)] du_1 du_2, \qquad \texttt{where } \tau = 2^{-r+1}.$$

The similar, but simpler approach applies also for $\mathbf{E}I_1(Y)$. Expectations $\mathbf{E}I_1(Z)$ and $\mathbf{E}I_2(Z)$ are estimated analogously. We deduce by (2.5) that $\mathbf{E}\omega''_{\delta_{r+1}}(X^{(1)}) = \mathsf{o}(1)$ as $\delta_{r+1} \to 0$, provided

$$\int_0^1 \int_0^1 (u_1 u_2)^{-2} [\sum_{i=1}^M f_i(u_1 u_2) \cdot \theta_i(u_1 u_2)] du_1 du_2 < \infty. \tag{2.10}$$

If $k \geq 2$ condition (2.10) is replaced by the following one

$$\int_0^1 \cdots \int_0^1 (u_1 \cdots u_k)^{-2} [\sum_{i=1}^M f_i(u_1 \cdots u_k) \cdot \theta_i(u_1 \cdots u_k)] du_1 \cdots du_2 < \infty.$$

But this condition is equivalent to (1.9), see, e.g., Klamkin (1976). Lemma is proved.

Proof of Theorem 4. It follows from Lemma above that

$$\forall \varepsilon > 0 \; \exists \delta > 0 \;\; : \quad \mathbf{E}\overline{\omega}''_\delta(X_n) < \varepsilon, \quad n \geq 1.$$

Now the statement of Theorem v4 follows from the Corollary of [B & W].

Proof of Theorem 2. Assume , without loss of generality that $F_1 = \ldots = F_M :=$ F and F is a probability measure with strictly increasing m.d.f. Let $k = 1$. When F is Lebesgue measure the theorem is proved in [B & F]. If F is arbitrary we use the transformation $X(t) \leftrightarrow X(F(t))$ to reduce the problem to the case when F is the Lebesgue measure.

For $k \geq 2$ in the proof we use induction on k. Let $F_1, ..., F_k$ denote m.d. functions of F. Following the proof of Theorem 15.7 of Billingsley (1968) we construct a sequence $\{X_n, \; n \geq 1\}$ of processes with sample paths in D_k which is weakly compact and the finitedimensional distributions of X_n converge to those of X. Fix integer $n \geq 1$ and consider the hyperplanes in R^k

$$H^{i,j} = \{\mathbf{t} = (t_1, ..., t_k), t_i = F_i^{-1}(j \cdot 2^{-n})\}, \; 0 \leq j \leq 2^n, \; 1 \leq i \leq k.$$

These hyperplanes divide the cube $[0,1]^k$ into rectangles $B_{\mathbf{s}} = [\mathbf{s}, \mathbf{t})$, where

$$\mathbf{s} = \mathbf{s}(j_1, ..., j_k) = (F_1^{-1}(j_1 2^{-n}), ..., F_k^{-1}(j_k \cdot 2^{-n})),$$

$$\mathbf{t} = (F_1^{-1}((j_1+1) \cdot 2^{-n}), ..., F_k^{-1}((j_k+1) \cdot 2^{-n})), \; 0 \leq j_r < 2^n, \; 1 \leq r \leq k.$$

Define the random process

$$X_n(\mathbf{u}) = \sum_{j_1,\dots,j_k} X(\mathbf{s}(j_1,\dots,j_k)) \cdot \mathbb{I}\{\mathbf{u} \in B_{\mathbf{s}(j_1,\dots,j_k)}\}, \quad \mathbf{u} \in [0,1]^k.$$

Here

$$B_{\mathbf{s}(j_1,\dots,j_k)} = \left\{(u_1,\dots,u_k) : F_i^{-1}(j_i \cdot 2^{-n}) \le u_i < F_i^{-1}((j_i+1)\cdot 2^{-n}),\ 1 \le i \le k\right\}.$$

By conditions (i) and (ii) of the theorem we get the convergence of the finitedimensional distributions of X_n to those of X. We argue as in the proof of Theorem 15.7 of Billingsley (1968) to show the weak compactness of the sequence $\{X_n,\ n \ge 1\}$ and also use the Corollary of [B & F] . In fact, it is enough to show that

$$\forall \varepsilon, \eta > 0\ \exists \delta > 0,\ \exists n_o :\ \forall n > n_o\ P(\ \overline{\omega}''_\delta(X_n) > \varepsilon\ ) < \eta.$$

To estimate this probability we follow the scheme of the proof of Lemma above. Once again for the sake of simplicity consider the case $k = 2$. Let $n \ge 1$ be arbitrary. Fix $i = 1$ and integer $m \ge 1$. Define the set S_m as above. Let $t \in S_{m+1}$. As in the proof of Lemma we need to estimate $\mathbf{E}\Delta(t_m^-, t, t_m^+)(X_n^{(1)})$. We follow the proof of Lemma but with $X_n^{(1)}$ instead of $X^{(1)}$ and we stop at the formula (2.4). By the induction hypothesis,

$$X_{n}{}^{(1)}_{(t_m^-)} - X_{n}{}^{(1)}_{(t)} \Rightarrow X^{(1)}_{(t_m^-)} - X^{(1)}_{(t)} \quad ; X_{n}{}^{(1)}_{(t)} - X_{n}{}^{(1)}_{(t_m^+)} \Rightarrow X^{(1)}_{(t)} - X^{(1)}_{(t_m^+)}, as\ n \to \infty$$

converges weakly in $D[0,1]$ (in D_{k-1} if $k \ge 2$ is arbitrary). Observe that

$$\|X_{n}{}^{(1)}_{(t)} - X_{n}{}^{(1)}_{(t_m^-)}\| \le \|X^{(1)}_{(t)} - X^{(1)}_{(t_m^-)}\| \quad \text{a.s.}$$

and

$$\|X_{n}{}^{(1)}_{(t)} - X_{n}{}^{(1)}_{(t_m^+)}\| \le \|X^{(1)}_{(t)} - X^{(1)}_{(t_m^+)}\| \quad \text{a.s.}$$

Thus

$$\Delta(t_m^-, t, t_m^+)(X_n^{(1)}) \le \|X^{(1)}_{(t_m^-)} - X^{(1)}_{(t)}\| \wedge \|X^{(1)}_{(t)} - X^{(1)}_{(t_m^+)}\|. \tag{2.11}$$

Note that up the moment we have defined only two one-parameter processes

$$X^{(1)}_{(t_m^-)} - X^{(1)}_{(t)} \qquad \texttt{and} \qquad X^{(1)}_{(t)} - X^{(1)}_{(t_m^+)}$$

with sample paths in $D[0,1]$ and we can replace $\Delta(t_m^-, t, t_m^+)(X_n^{(1)})$ in (2.4) by (2.11) and continue the calculation in the proof of Lemma but with $X^{(1)}_{(t_m^-)} - X^{(1)}_{(t)}$ and $X^{(1)}_{(t)} - X^{(1)}_{(t_m^+)}$ instead of ${X_n}^{(1)}_{(t_m^-)} - {X_n}^{(1)}_{(t)}$ and ${X_n}^{(1)}_{(t)} - {X_n}^{(1)}_{(t_m^+)}$ .The subsequent steps of proofs of the theorem and the Lemma coincide. We have that for each $\varepsilon > 0$ there exists a $\delta > 0$ independent of n such that $\mathbf{E}\overline{\omega}''_\delta(X_n) < \varepsilon$ for each $n \geq 1$. Theorem 2 is proved.

Proof of theorem 1. First we prove that there exists a centered sample continuous Gaussian random process on $[0,1]^k$ with the same covariance as X. For each $\mathbf{s} = (s_1, ..., s_k)$ and $\mathbf{t} = (t_1, ..., t_k)$ we have

$$\mathbf{E}(X(\mathbf{t}) - X(\mathbf{s}))^2 \leq k(\mathbf{E}[\mathbf{\Delta}^{(1)}_{(s_1,t_1]} X(t_1, ..., t_k)]^2 +$$

$$+\mathbf{E}[\mathbf{\Delta}^{(2)}_{(s_2,t_2]} X(s_1, t_2, ..., t_k)]^2 + ... + \mathbf{E}[\mathbf{\Delta}^{(k)}_{(s_k,t_k]} X(s_1, ..., s_{k-1}, t_k)]^2) \leq$$

$$\leq k \cdot \sum_{i=1}^{k} g^{2/q}(G_i(t_i) - G_i(s_i)).$$

Here $G_i,\ 1 \leq i \leq k$ denote the marginal distribution functions of measure G . Assume without loss of generality that G is probability measure with continuous and strictly increasing marginal d.f. We have

$$\mathbf{E}(X(\mathbf{t}) - X(\mathbf{s}))^2 \leq c \cdot g^{2/q}(\max_{1 \leq i \leq k} \mid G_i(t_i) - G_i(s_i) \mid).$$

If we denote $\overline{F}(t_1, ..., t_k) = (G_1^{-1}(t_1), ..., G_k^{-1}(t_k)) : [0,1]^k \to [0,1]^k$ then

$$\mathbf{E}[X(\overline{F}(\mathbf{t})) - X(\overline{F}(\mathbf{s}))]^2 \leq c \cdot g^{2/q}(\|\mathbf{t} - \mathbf{s}\|).$$

This inequality together with (1.5) yields that the Gaussian random process with the covariance of X is sample continuous on $[0,1]^k$, see Fernique (1964).

Note that the random process X satisfies the conditions of Theorem 2. The proof of the CLT goes along the lines of the proof of Theorem 2 in Bloznelis and

Paulauskas (1993) only now we use Theorem 4 instead of Theorem 1.3 of [B & F] and Lemmas 5 and 6 instead of Lemmas 1 and 2 in Bloznelis and Paulauskas (1993). Theorem 1 is proved.

Proof of Example 1.Since the proof goes along the lines of the proof in the case $k = 1$ (see Bloznelis and Paulauskas (1993), Fernique (1993) and Hahn (1977)),only the calculations are more complicated, we shall give the skech of the proof. To simplify the writing we consider the case $k = 2$ and $p = 2$. At first step we construct the process $\tilde{X}$ on $[0,1]^2$ satisfying (1.2) with a function which satisfies (1.6) and (1.7). Let $A_{kj} = [2^{-k}, 2^{-k+1}) \times [2^{-j}, 2^{-j+1})$ and

$$I_n = (\bigcup_{1\le j\le n} A_{nj}) \cup (\bigcup_{1\le j< n} A_{jn}),\ n \ge 1.$$

Define a sequence $a_n = \sum_{l,m=1}^{n} f^{1/2}(2^{-(l+m)})2^{(l+m)/2}$. Due to (1.6) $a_n \to \infty$.Now define a function φ on $T = [0,1)^2$ by relations: $\varphi(\mathbf{t}) = 0$ if $\mathbf{t} \in LB(T)$, $\varphi(\mathbf{t}) = a_n$ for $\mathbf{t} \in I_n$. Extend this function to $[0,2)^2$ setting $\varphi(\mathbf{t}) = \varphi(\mathbf{t} + (1,1)) = \varphi(\mathbf{t} + (0,1)) = \varphi(\mathbf{t} + (1,0))$ for all $\mathbf{t} \in [0,1)^2$. As a probability space $(\Omega, \mathcal{A}, P)$ take the set $[0,1)^2$ with the Lebesgue measure and define $\tilde{X}(\mathbf{t},\omega) = \varphi(\mathbf{t}+\omega)$, $\mathbf{t} \in [0,1]^2$. It is possible to show (here we use (1.7)) that $\tilde{X}$ satisfies the condition (1.2) and also the analogous condition on the lower boundary of $[0,1]^2$,which is discussed after the formulation of Theorem 1. Obviously the process $\tilde{X}$ has no modification in D_2. The second step is to modify the process $\tilde{X}$ in order to get the process in D_2, which fails to satisfy CLT. Take a function $M : \Omega \to R^+$ such that

$$\lim_{n\to\infty} nP(\omega : M(\omega) \ge \sqrt{n}) = \infty$$

(for example, we can take $M(\omega) = (\omega_1\omega_2)^{-\alpha}$ with any $1/2 \le \alpha \le 1$). Now define

$$\bar{X}(\mathbf{t},\omega) = \tilde{X}(\mathbf{t},\omega) \text{ if } \tilde{X}(\mathbf{t},\omega) \le M(\omega) \text{ and } \bar{X}(\mathbf{t},\omega) = M(\omega) \text{ if } \tilde{X}(\mathbf{t},\omega) \ge M(\omega).$$

Finally, symmetrizing on the space $(\Omega, \times\{0,1\}, \mathbf{P})$, where $\mathbf{P} = P \times (2^{-1}\delta_0 + 2^{-1}\delta_1)$, we get the process

$$X(\mathbf{t},\bar{\omega}) = X(\mathbf{t},\omega\times l) = \begin{cases} \bar{X}(\mathbf{t},\omega), & \text{l=0,} \\ -\bar{X}(\mathbf{t},\omega), & \text{l=1} \end{cases}$$

with almost all sample paths in D_2. Repeating the proof of Theorem 1 in Hahn (1977) one can show that $X \notin CLT(D_2)$. It remains to note that

$$\mathbf{E}(|X(B)| \wedge |X(C)|)^2 = \mathbf{E}(|\bar{X}(B)| \wedge |\bar{X}(C)|)^2 \leq \mathbf{E}(|\tilde{X}(B)| \wedge |\tilde{X}(C)|)^2.$$

Proof of Proposition 7. By the CLT, sequence of random processes $\{S_n(t) = n^{-1/2}(X_1(t) + \cdots + X_n(t)),\ n \geq 1\}$ is stochastically bounded, i.e.,

$$\forall \varepsilon > 0\ \exists H > 0\ : P(\|S_n\| > H) < \varepsilon, \quad \forall n \geq 1.$$

We argue as in Kuelbs (1973) p.p. 165-167 to show that given $\varepsilon, \delta > 0$ there is an integer $r \geq 1$ such that if $h = 2^{-r}$, then

$$P(\sup_{|s'-s''| \leq h} \|Z_n(s', \cdot) - Z_n(s'', \cdot)\| > \delta) < \varepsilon, \quad \forall n \geq 1. \tag{2.12}$$

Now fix $h = 2^{-r}$ so that (2.12) holds and consider the sequences of ran dom processes

$$\{Z_n(i/(2^r), t),\ t \in [0,1]^k,\ n \geq 1\}, \quad 0 \leq i \leq 2^r.$$

For each i the sequence $\{Z_n(i/(2^r), t), t \in [0,1]^k,\ n \geq 1\}$ converge weakly in D_k to a sample continuous Gaussian r. process because $X \in CLT(D_k)$ and S_n converge weakly to a sample continuous Gaussian r. process Y. By Lemma 5 we have that given $\varepsilon, \delta > 0$ there is an integer n_o and $\tau > 0$ such, that if $n > n_o$, then

$$P(\max_{0 \leq i \leq 2^r} \sup_{\|\mathbf{t}'-\mathbf{t}''\| < \tau} | Z_n(i/(2^r), \mathbf{t}') - Z_n(i/(2^r), \mathbf{t}'') | > \delta) < \varepsilon. \tag{2.13}$$

Combining (2.12) and (2.13) we obtain

$$P(\sup_{\|u'-u''\| < \tau \wedge h} | Z_n(u') - Z_n(u'') | > 3\delta) < 2\varepsilon,$$

where $u' = (s', \mathbf{t}'),\ u'' = (s'', \mathbf{t}'') \in [0,1] \times [0,1]^k$. Proposition 7 now follows since the finitedimensional distributions of Z_n converge to the corresponding finitedimensional distributions of W_Y.

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